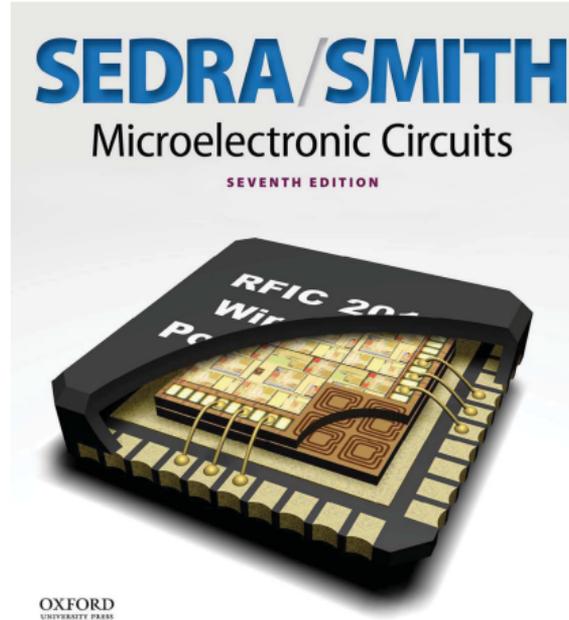


# Diode models

Kizito NKURIKIYEYEU, Ph.D.

# Readings

- Section 4.3 on pages 193-195



<sup>1</sup> Readings are based on Sedra & Smith (2014), Microelectronic Circuits 7th edition.

<sup>2</sup> Bold reading section are mandatory. Other sections are suggested but not required readings

# Background

How would you find  $I_D$  and  $V_D$  for the circuit in Fig. 2

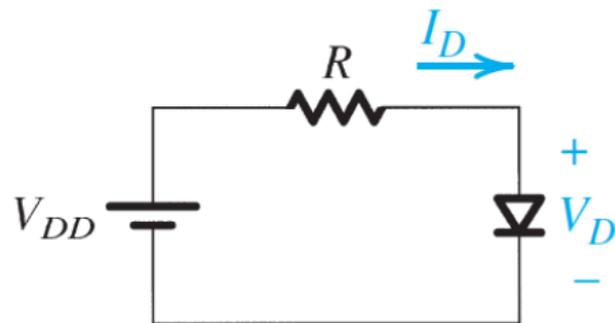
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**FIG 1.** Illustrative diode circuit

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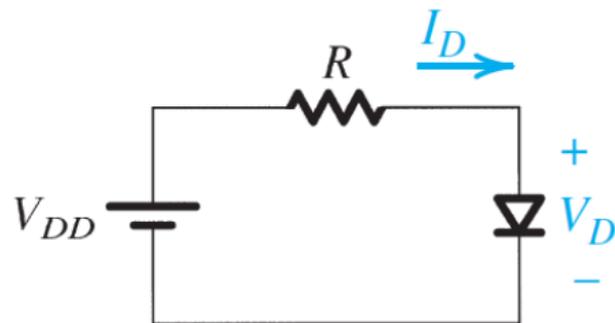
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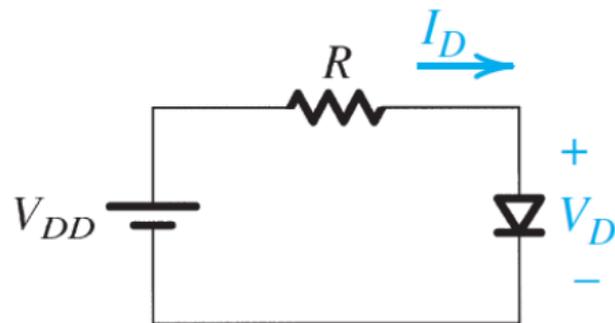


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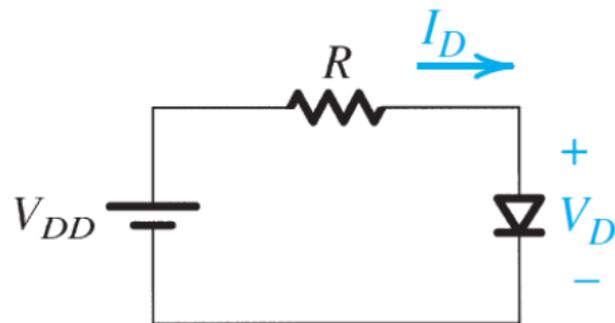
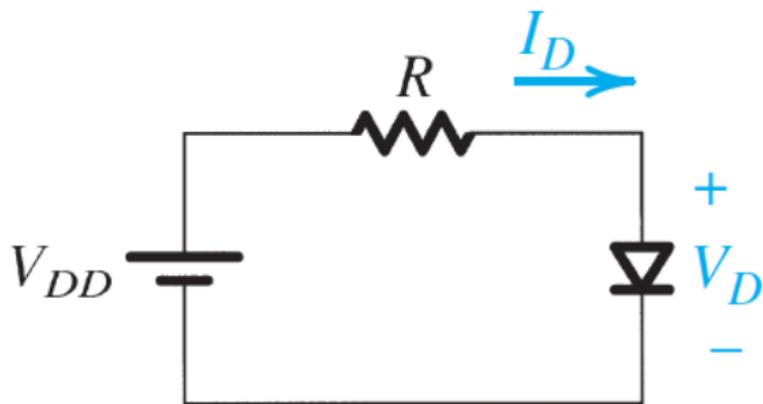


FIG 1. Illustrative diode circuit

# Background

- **model** —a mathematical description or electrical equivalent circuit that represents the behavior of a device or system
- In this lecture, we shall learn simplified diode models that are suited for circuit analysis:
  - Exponential model
  - Constant voltage-drop model
  - ideal diode model
  - small-signal model



**FIG 2.** Illustrative diode circuit

# The exponential diode model

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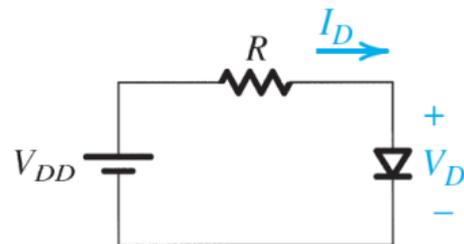
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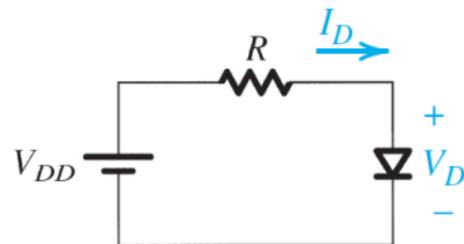
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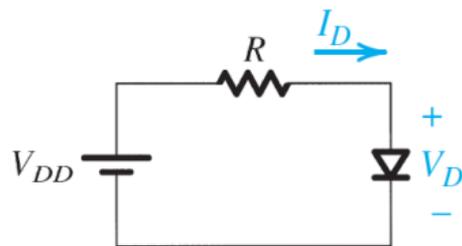
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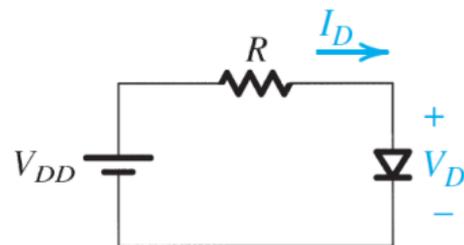
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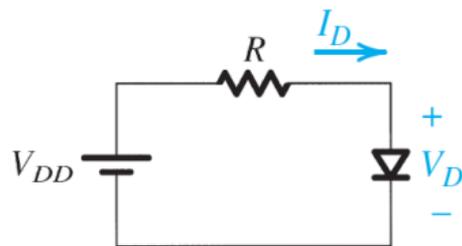
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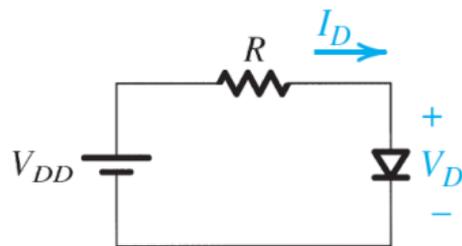
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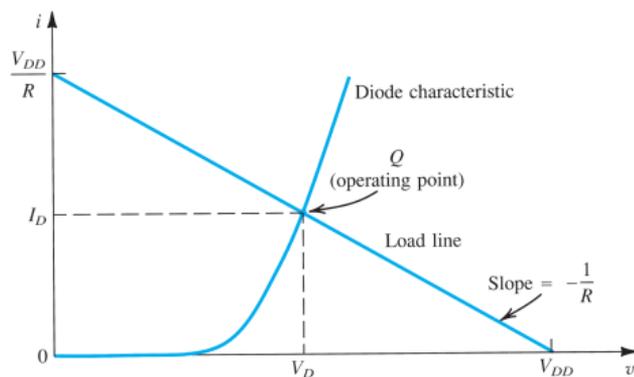
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# Graphical analysis

- Plot the relationships of **Equation (3)** and **Equation (4)** on an  $i$ - $v$  plane.
- The solution is the **Q-point**—which is the coordinates of the point of intersection of the two graphs
- The Q point is also known as the operating point, the bias point, or quiescent point<sup>1</sup>
- The Q-point is the steady-state voltage or current at a specified terminal of an active diode with no input signal applied<sup>2</sup>



**FIG 3.** Illustration of the graphical analysis method using the exponential diode model

<sup>1</sup> The graphical analysis is only used for visualization of simple circuit but it is tedious for complex circuit; thus, is seldomly used in practice

<sup>2</sup> For details, please refer to the [The Importance of the Q Point of a Diode to Circuit Functionality](#)

<sup>3</sup> <https://en.wikipedia.org/wiki/Biasing>

# Iterative analysis<sup>1</sup>

**EXAMPLE**—Find  $I_D$  and  $V_D$  for the circuit in Section 1 when  $V_{DD} = 5V$  and  $R = 1k$ . Assume that the diode has a current of  $I_D = 1mA$  at a voltage of  $V_D = 0.7V$ .

- $I_D$  is found by KVL

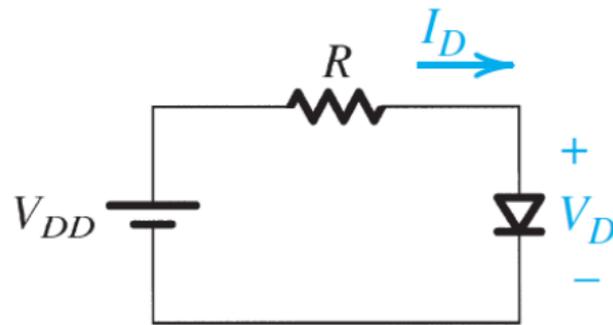
$$I_D = \frac{V_{DD} - V_D}{R} = \frac{5V - 0.5V}{1k} = 4.3mA \quad (5)$$

- $V_D$  is deducted from

$$V_2 - V_1 = 2.3V_T \cdot \log(I_2/I_1) \quad (6)$$

Since  $2.3V_T = 60mV$ , then

$$V_2 = V_1 + 0.06 \cdot \log(I_2/I_1) \quad (7)$$



<sup>1</sup> See detailed algebraic solution at [https://en.wikipedia.org/wiki/Diode\\_modelling#Iterative\\_solution](https://en.wikipedia.org/wiki/Diode_modelling#Iterative_solution)

# Iterative analysis<sup>1</sup>

- The first iteration assumes  $V_1 = 0.7V$ ,  $I_1 = 1mA$  and  $I_2 = 4.3mA$  that we calculated earlier. Thus,  $V_D = 0.738V$
- The second iteration goes through the same process

$$I_D = \frac{V_{DD} - V_D}{R} = \frac{5V - 0.738V}{1k} = 4.262mA \quad (8)$$

$$\begin{aligned} V_2 &= V_1 + 0.06 \cdot \log(I_2/I_1) \\ &= 0.738V + 0.06 \cdot \log(4.262/4.3) \\ &= 0.738V \end{aligned} \quad (9)$$

- The iteration can continue but the second iteration yielded values close to the first iteration, there is no reason to continue any further.
- Thus,  $I_D \approx 4.262mA$  and  $V_D \approx 0.738V$

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# Iterative analysis—Practicality

- This method is simple and very accurate
- However, it is very slow and not practical. Circuit design requires evaluating various possibility before making a suitable design
- In practice, analog circuit design is something of an art. Although it is possible to predict the behavior of a very simple circuit mathematically, there are so many factors to consider in a more complicated circuit that the calculations become impossibly convoluted
- It is best to use less accurate methods and verify the design with computer analysis tools such as SPICE

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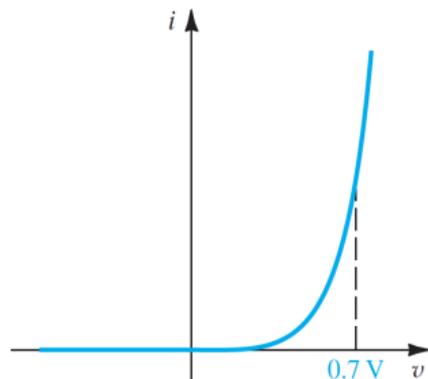
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- The simplest and most widely used diode model in the initial phases of analysis and design
- Since the forward-conducting diode has a voltage drop that varies in a relatively narrow range (usually between 0.6 to 0.8 V), assumes that the slope of the i-v curve is vertical at  $V_D = 0.7\text{ V}$

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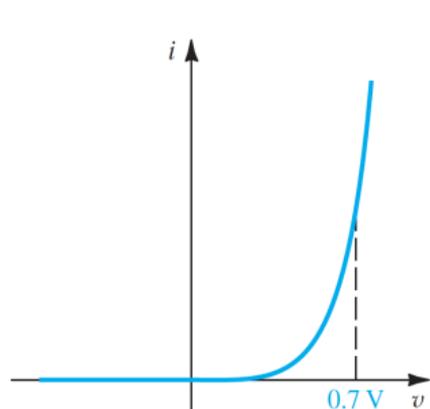
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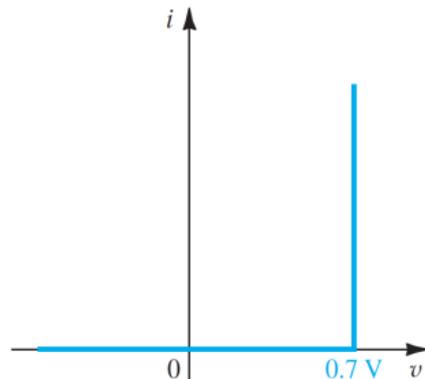
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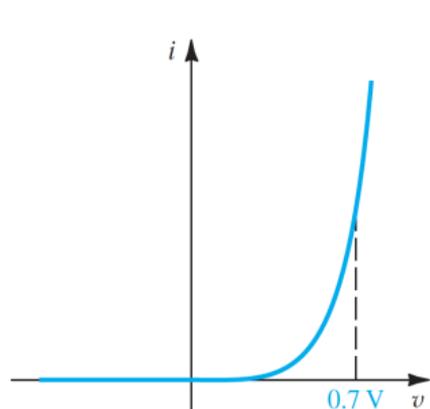
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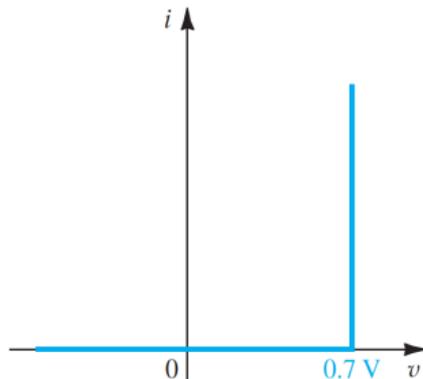
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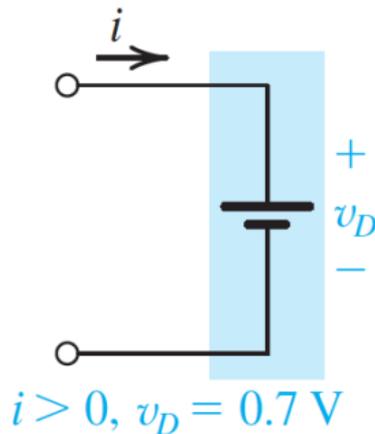
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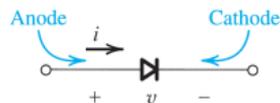


(c) the resulting model of the forward-conducting diodes

**FIG 4.** Development of the diode constant-voltage-drop model:

# The Ideal-Diode Model

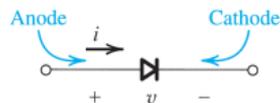
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- In this case, we may neglect the diode voltage drop altogether while calculating the diode current.
- In summary, the ideal diode model assumes that the slope of i-v curve is vertical at  $V_D = 0V$



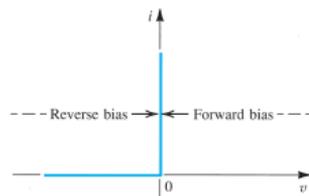
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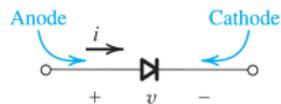
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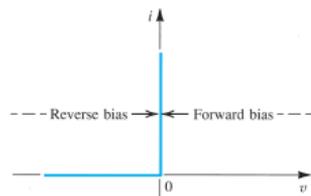
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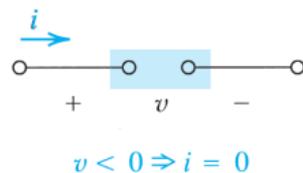
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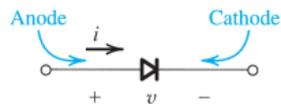
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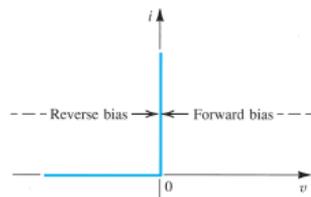
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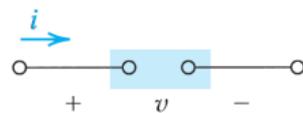
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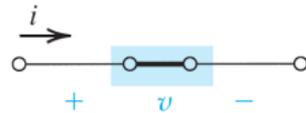


(b) i-v characteristic



$$v < 0 \Rightarrow i = 0$$

(c) equivalent circuit in the reverse direction



$$i > 0 \Rightarrow v = 0$$

(d) equivalent circuit in the forward direction

**FIG 5.** The ideal diode model

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