APPENDIX H

DESIGN OF STAGGER-TUNED AMPLIFIERS

As mentioned in Section 17.11.7, a much better overall response of a tuned amplifier is obtained by stagger-tuning (as opposed to synchronous tuning) the individual stages, as illustrated in Fig. 17.46 which is repeated here as Fig. H.1. Stagger-tuned amplifiers are usually designed so that the overall response exhibits *maximal flatness* around the center-frequency f_0. Such a response can be obtained by transforming the response of a maximally flat (Butterworth) low-pass filter up the frequency axis to ω_0. We show here how this can be done.

The transfer function of a second-order bandpass filter can be expressed in terms of its poles as

$$T(s) = \frac{a_1 s}{\left(s + \frac{\omega_0}{2Q} - j\omega_0\sqrt{1 - \frac{1}{4Q^2}}\right)\left(s + \frac{\omega_0}{2Q} + j\omega_0\sqrt{1 - \frac{1}{4Q^2}}\right)} \tag{H.1}$$

For a narrow-band filter, $Q \gg 1$, and for values of s in the neighborhood of $+j\omega_0$ (see Fig. H.2b), the second factor in the denominator is approximately $(s + j\omega_0 \simeq 2s)$. Hence Eq. (H.1) can be approximated in the neighborhood of $j\omega_0$ by

$$T(s) \simeq \frac{a_1/2}{s + \omega_0/2Q - j\omega_0} = \frac{a_1/2}{(s - j\omega_0) + \omega_0/2Q} \tag{H.2}$$

This is known as the **narrow-band approximation**.[1] Note that the magnitude response, for $s = j\omega$, has a peak value of $a_1 Q/\omega_0$ at $\omega = \omega_0$, as expected.

Now consider a first-order low-pass network with a single pole at $p = -\omega_0/2Q$ (we use p to denote the complex frequency variable for the low-pass filter). Its transfer function is

$$T(p) = \frac{K}{p + \omega_0/2Q} \tag{H.3}$$

where K is a constant. Comparing Eqs. (H.2) and (H.3) we note that they are identical for $p = s - j\omega_0$ or, equivalently,

$$s = p + j\omega_0 \tag{H.4}$$

[1]The bandpass response is *geometrically symmetrical* around the center frequency ω_0. That is, each pair of frequencies ω_1 and ω_2 at which the magnitude response is equal are related by $\omega_1\omega_2 = \omega_0^2$. For high Q, the symmetry becomes almost *arithmetic* for frequencies close to ω_0. That is, two frequencies with the same magnitude response are almost equally spaced from ω_0. The same is true for higher-order bandpass filters designed using the transformation presented in this section.

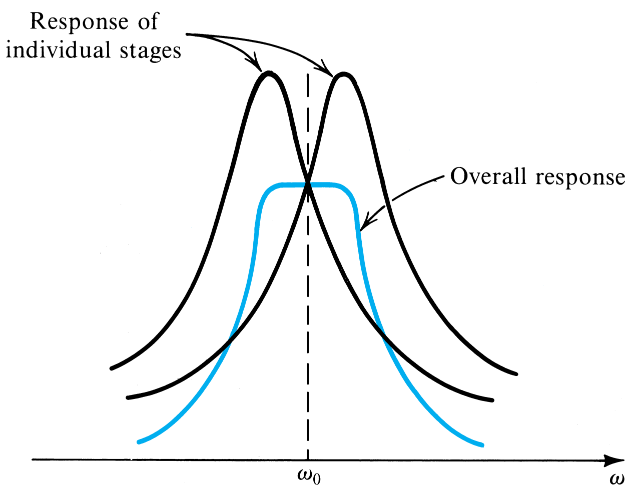

Figure H.1 Stagger-tuning the individual resonant circuits can result in an overall response with a passband flatter than that obtained with synchronous tuning (Fig. 17.48).

This result implies that the response of the second-order bandpass filter *in the neighborhood of its center frequency* $s = j\omega_0$ is identical to the response of a first-order low-pass filter with a pole at $(-\omega_0/2Q)$ *in the neighborhood of* $p = 0$. Thus the bandpass response can be obtained by shifting the pole of the low-pass prototype and adding the complex-conjugate pole, as illustrated in Fig. H.2(b). This is called a **lowpass-to-bandpass transformation** for *narrow-band* filters.

The transformation $p = s - j\omega_0$ can be applied to low-pass filters of order greater than one. For instance, we can transform a maximally flat, second-order low-pass filter $(Q = 1/\sqrt{2})$ to obtain a maximally flat bandpass filter. If the 3-dB bandwidth of the bandpass filter is to be B rad/s, then the low-pass filter should have a 3-dB frequency (and thus a pole frequency) of $(B/2)$ rad/s, as illustrated in Fig. H.3. The resulting fourth-order bandpass filter will be a stagger-tuned one, with its two tuned circuits (refer to Fig. 16.48) having

$$\omega_{01} = \omega_0 + \frac{B}{2\sqrt{2}} \qquad B_1 = \frac{B}{\sqrt{2}} \qquad Q_1 \simeq \frac{\sqrt{2}\omega_0}{B} \tag{H.5}$$

$$\omega_{02} = \omega_0 - \frac{B}{2\sqrt{2}} \qquad B_2 = \frac{B}{\sqrt{2}} \qquad Q_2 = \frac{\sqrt{2}\omega_0}{B} \tag{H.6}$$

Note that for the overall response to have a normalized center-frequency gain of unity, the individual responses have to have equal center-frequency gains of $\sqrt{2}$, as shown in Fig. H.3(d).

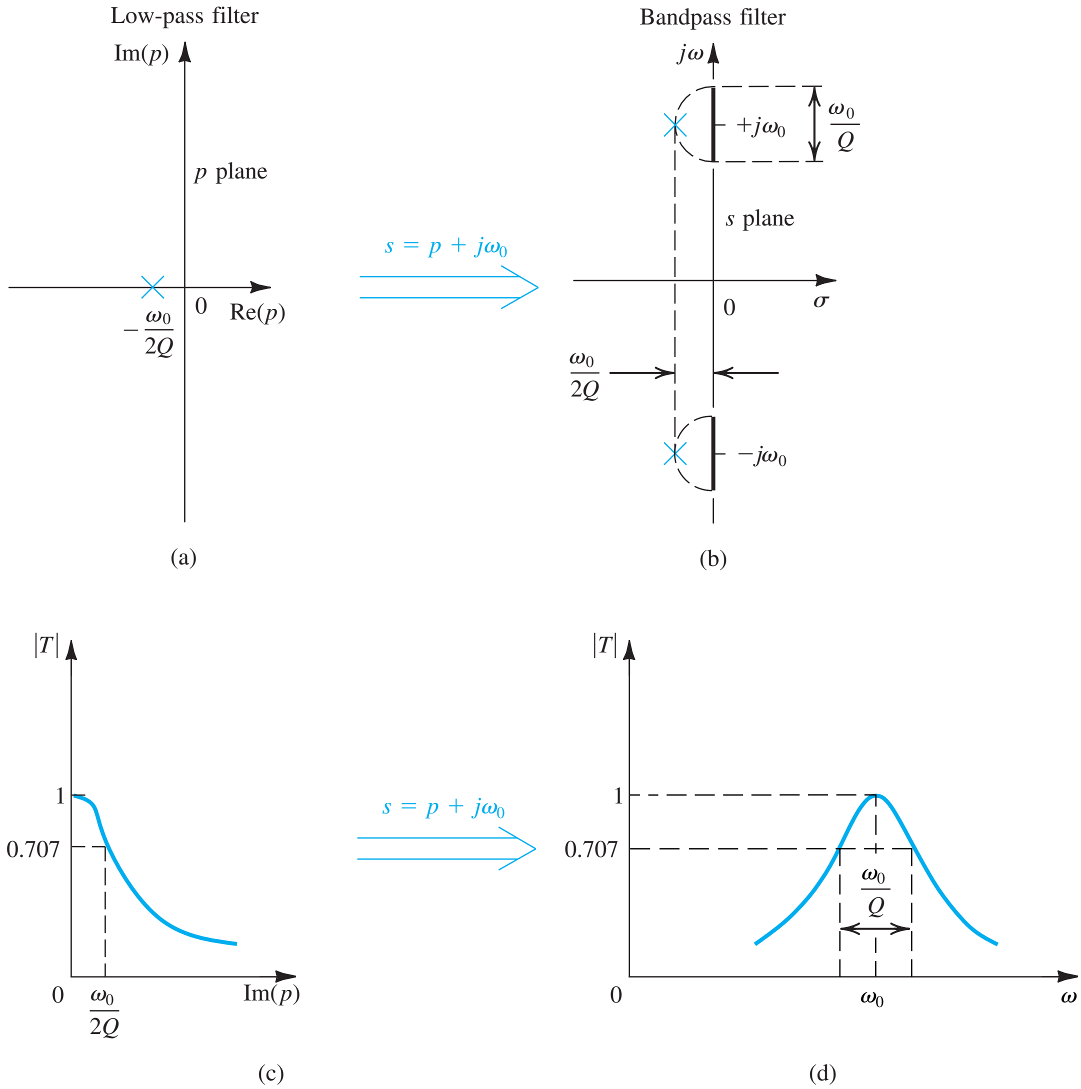

Figure H.2 Obtaining a second-order narrow-band bandpass filter by transforming a first-order low-pass filter. **(a)** Pole of the first-order filter in the p plane. **(b)** Applying the transformation $s = p + j\omega_0$ and adding a complex-conjugate pole results in the poles of the second-order bandpass filter. **(c)** Magnitude response of the first-order low-pass filter. **(d)** Magnitude response of the second-order bandpass filter.

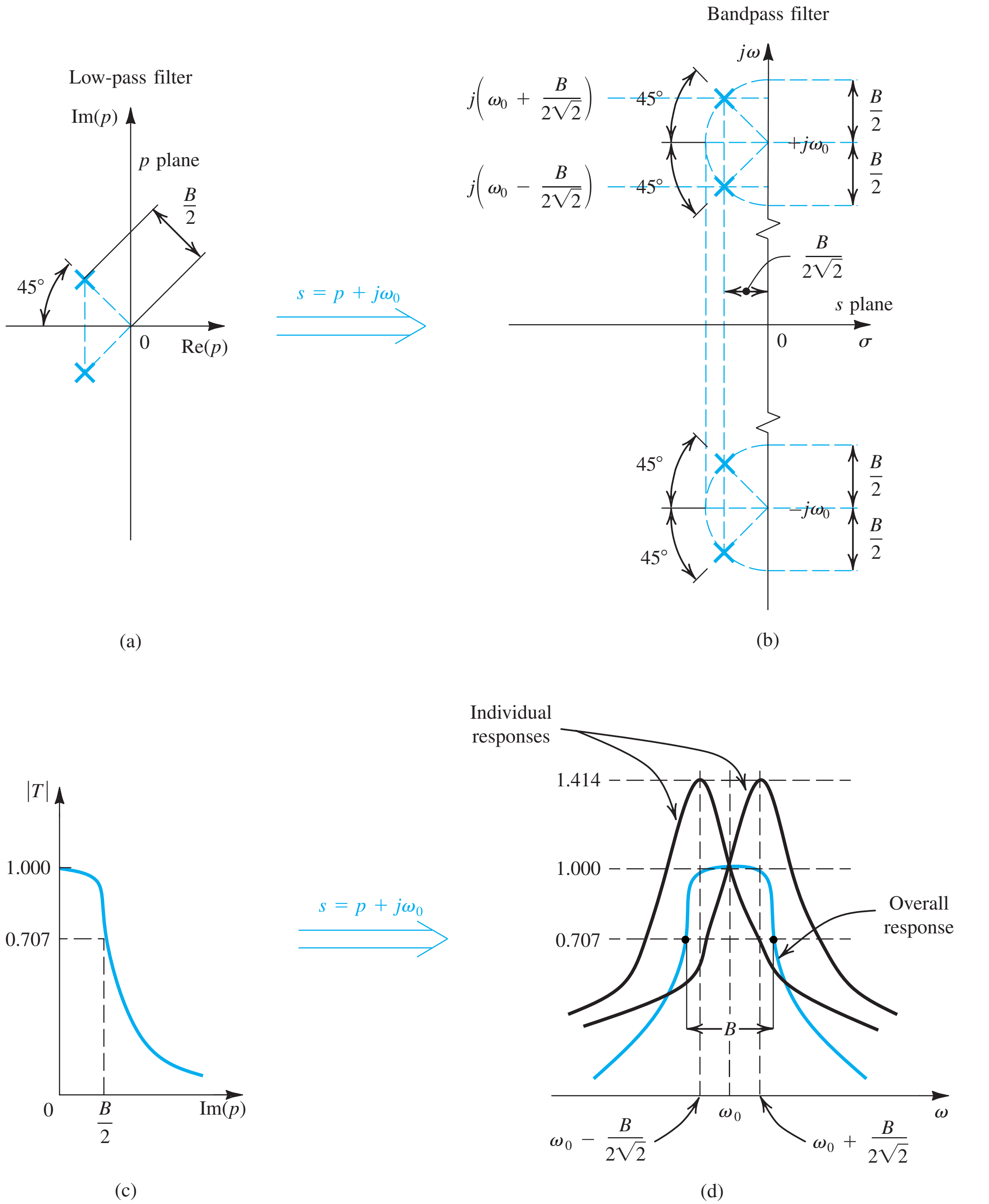

Figure H.3 Obtaining the poles and the frequency response of a fourth-order stagger-tuned, narrow-band bandpass amplifier by transforming a second-order low-pass, maximally flat response.

EXERCISES

D H.1 A stagger-tuned design for the IF amplifier specified in Exercise 17.36 is required. Find f_{01}, B_1, f_{02}, and B_2. Also give the value of C and R for each of the two stages. (Recall that 3-μH inductors are to be used.)

Ans. 10.77 MHz; 141.4 kHz; 10.63 MHz; 141.4 kHz; 72.8 pF; 15.5 kΩ; 74.7 pF; 15.1 kΩ

H.2 Using the fact that the voltage gain at resonance is proportional to the value of R, find the ratio of the gain at 10.7 MHz of the stagger-tuned amplifier designed in Exercise H.1 and the synchronously tuned amplifier designed in Exercise 17.36. (*Hint:* For the stagger-tuned amplifier, note that the gain at ω_0 is equal to the product of the gains of the individual stages at their 3-dB frequencies.)

Ans. 2.42

PROBLEMS

***H.1** This problem investigates the selectivity of maximally flat stagger-tuned amplifiers derived in the manner illustrated in Fig. H.3.

(a) The low-pass maximally flat (Butterworth) filter having a 3-dB bandwidth $B/2$ and order N has the magnitude response

$$|T| = 1 \Big/ \sqrt{1 + \left(\frac{\Omega}{B/2}\right)^{2N}}$$

where $\Omega = \text{Im}(p)$ is the frequency in the low-pass domain. (This relationship can be obtained using the information provided in Section 17.3 on Butterworth filters.) Use this expression to obtain for the corresponding bandpass filter at $\omega = \omega_0 + \delta\omega$, where $\delta\omega \ll \omega_0$, the relationship

$$|T| = 1 \Big/ \sqrt{1 + \left(\frac{\delta\omega}{B/2}\right)^{2N}}$$

(b) Use the transfer function in (a) to find the attenuation (in decibels) obtained at a bandwidth of $2B$ for $N = 1$ to 5. Also find the ratio of the 30-dB bandwidth to the 3-dB bandwidth for $N = 1$ to 5.

****H.2** Consider a sixth-order, stagger-tuned bandpass amplifier with center frequency ω_0 and 3-dB bandwidth B. The poles are to be obtained by shifting those of the third-order maximally flat low-pass filter, given in Fig. 17.10(c). For each of the three resonant circuits, find ω_0, the 3-dB bandwidth, and Q.