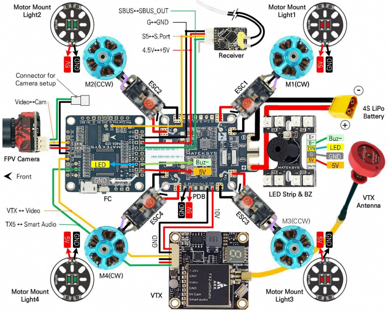
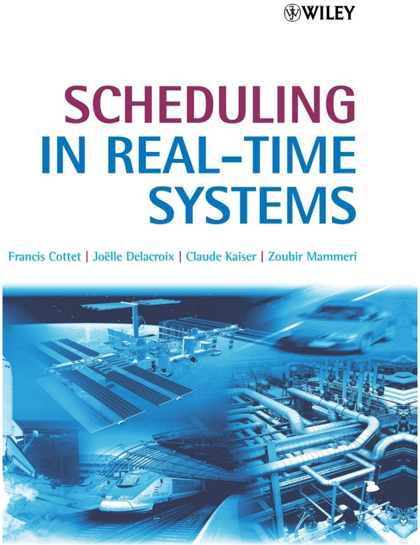


Scheduling of independent tasks

Kizito NKURIKIYEYEZU, Ph.D.

Readings

- Read Chapter 2, section 2.1 (pages 23-33) of Cottet et al. (2002). Scheduling in Real-Time Systems. Skip other sections!
- Topics
  - rate monotonic
  - inverse deadline
  - earliest deadline first
  - least laxity first
  - On-line scheduling



[1] Readings are based on Cottet, F., Delacroix, J., Mammeri, Z., & Kaiser, C. (2002). Scheduling in Real-Time Systems. Wiley.

Review

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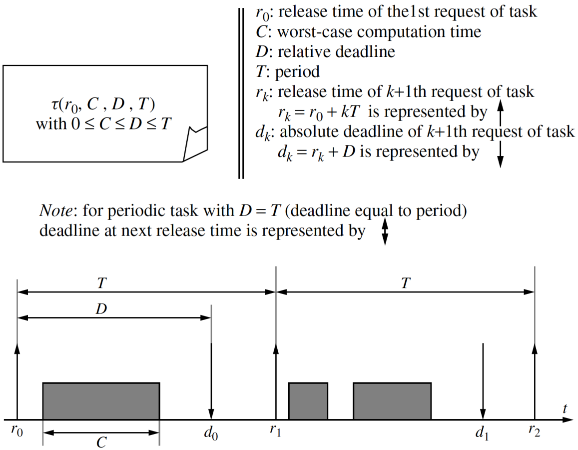
Real-time task model

- r_i, task release time, i.e. the execution request time.



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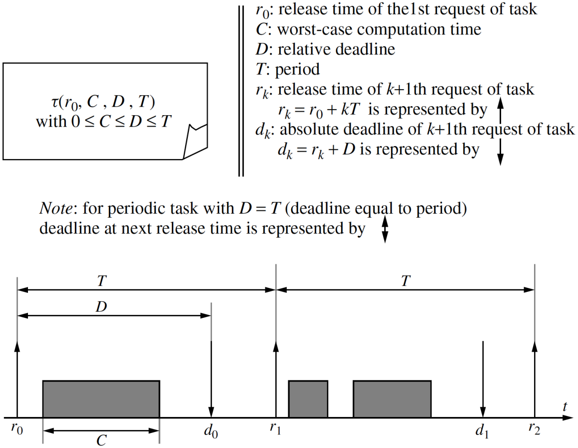

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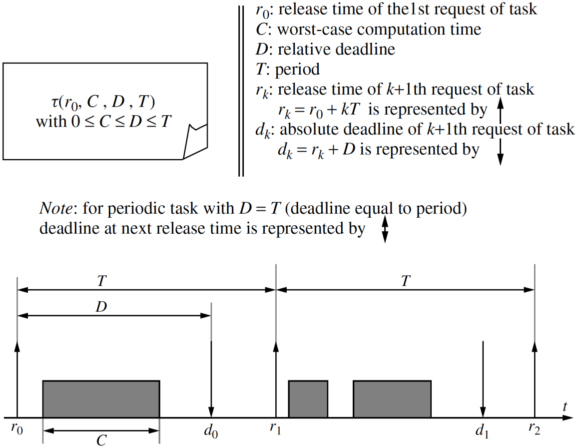

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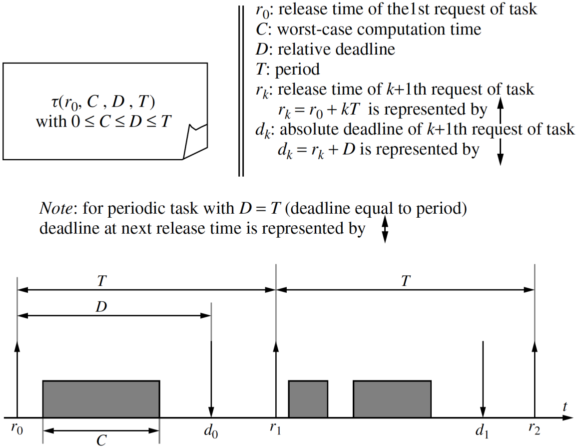

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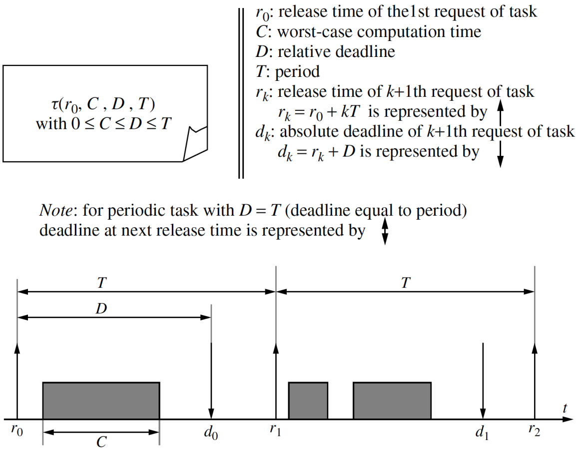
- Absolute deadline $d_i = r_i + D_i$—transgression of the absolute deadline causes a timing fault.



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Relative Deadline vs Period

When we have a task set, we say that the task set is with

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- arbitrary deadline when the relative deadline D_i could be larger than the period T_i for some task τ_i

Sporadic and Periodic Tasks

- For periodic taks
  - Syncronous system—each task τ_i has a phase of 0, i.e., $\phi_i = 0$

- Hyperperiod: Least common multiple (LCM) of T_i
- Task utilization of task

$$u_i = \frac{C_i}{T_i} \tag{1}$$

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simple rule—that assigns priorities according to temporal parameters of tasks.

- static—the priority is fixed. the priorities are assigned to tasks before execution and do not change over time. For example:
  - rate monotonic (Liu and Layland, 1973)[1]

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- For two tasks (i.e., $n = 2$), the upper bounds on utilization is (Equation (4))

$$n\left(2^{\frac{1}{2}} - 1\right) = 2\left(\sqrt{2} - 1\right) = 0.828 \tag{4}$$

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- As a general rule, when $n > 10$, the RMS can meet its deadlines if $U < 70\%$

Rate monotonic scheduling—Example 1

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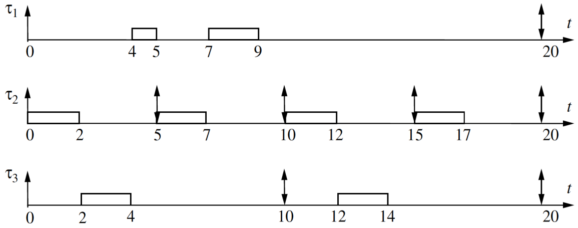

FIG 2. Example of a rate monotonic schedule with three periodic tasks: $\tau_1(0, 3, 20, 20)$, $\tau_2(0, 2, 5, 5)$ and $\tau_3(0, 2, 10, 10)$

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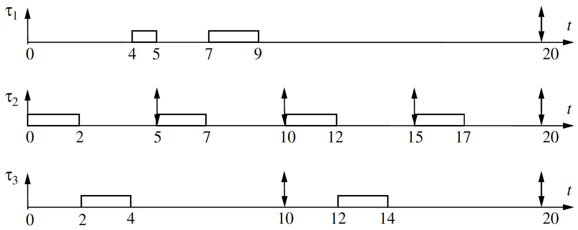

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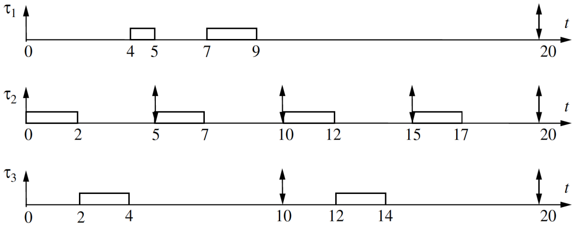

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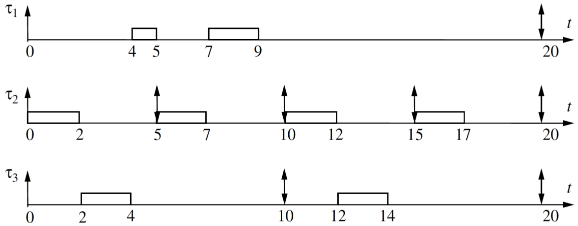
- After τ_2 completes, the lowest priority task, τ_1, executes until $t = 5$



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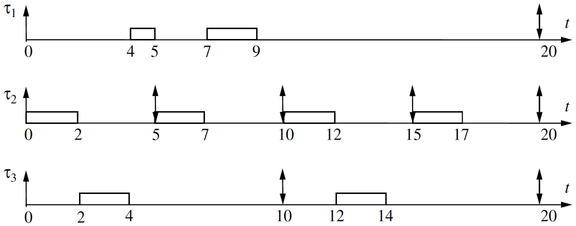

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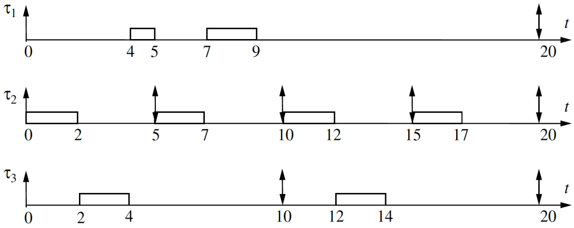

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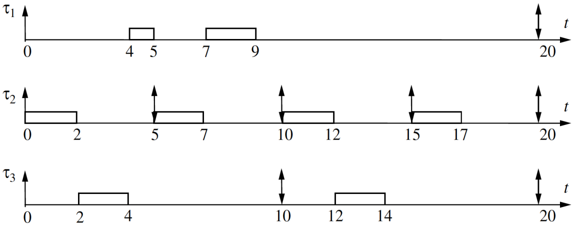

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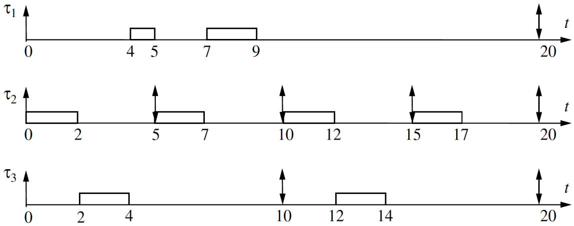

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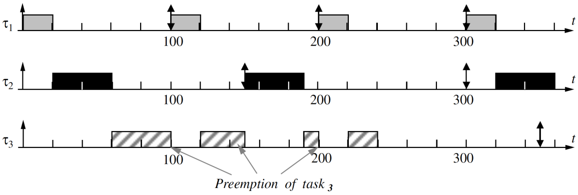
The three tasks meet their deadline since the utilization factors

$$U = \frac{3}{20} + \frac{2}{5} + \frac{2}{10} = 0.75 \leq 3(2^{\frac{1}{3}} - 1) = 0.779 \tag{6}$$

Rate monotonic scheduling —Example 2

- In this example, we have a set of three periodic tasks for which the relative deadline is equal to the period



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Rate monotonic scheduling —Example 2

- In this example, we have a set of three periodic tasks for which the relative deadline is equal to the period
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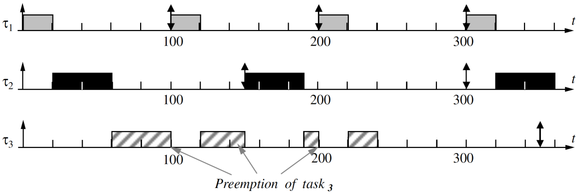

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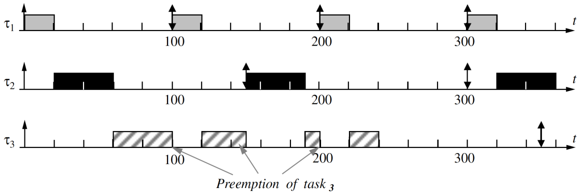

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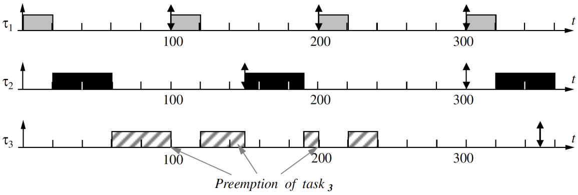

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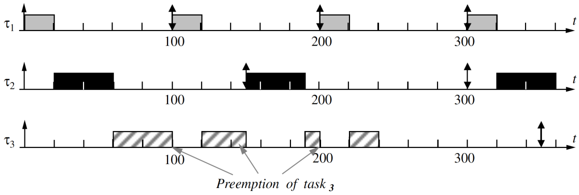

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The processor utilization factor is:

$$U = \frac{20}{100} + \frac{40}{150} + \frac{100}{350} = 0.75 < 3 \cdot (\sqrt[3]{2} - 1) = 0.779 \tag{7}$$

So this task set is schedulable. All the three tasks meet their deadlines.

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- For smaller values of n or in cases where U is close to this estimate, the calculated utilization bound should be used.

Inverse (monotonic) deadline algorithm

Deadline-monotonic scheduling

summary—Deadline-monotonic priority assignment is a priority assignment policy used with fixed-priority pre-emptive scheduling[5]

- Allows a weakening of the condition which requires equality between periods and deadlines in static-priority schemes.

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- The task with the shortest relative deadline is assigned the highest priority[6]
- For an arbitrary set of n tasks with deadlines shorter than periods, a sufficient condition is given in Equation (8)

$$U = \sum_{i=1}^{n} \frac{C_i}{D_i} \leq n \cdot \left(2^{\frac{1}{n}} - 1\right) \tag{8}$$

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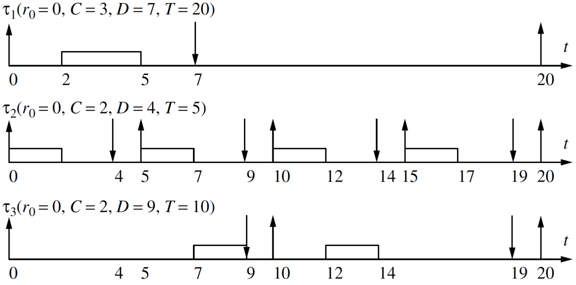

FIG 5. Inverse deadline schedule for a set of three periodic tasks τ_1 $(r_0 = 0, C = 3, D = 7, T = 20)$, $\tau_2(r_0 = 0, C = 2, D = 4, T = 5)$ and $\tau_3(r_0 = 0, C = 2, D = 9, T = 10)$

$$U = \frac{3}{7} + \frac{2}{4} + \frac{2}{9} = 1.15 > 3(\sqrt[3]{2} - 1) = 0.779 \qquad (9)$$

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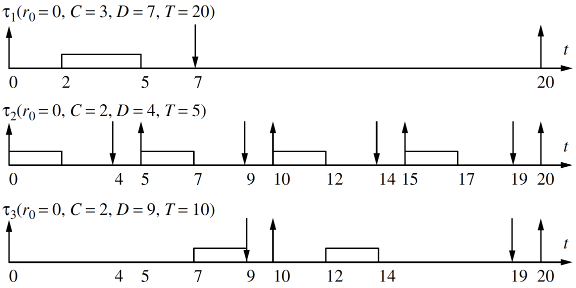

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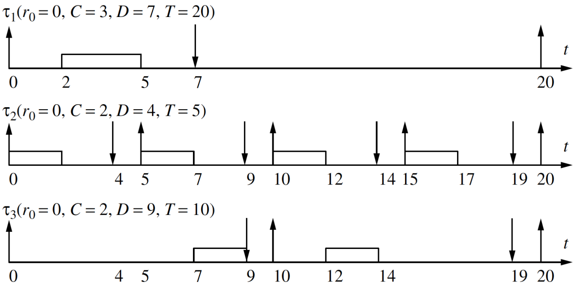

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- A hybrid task set is schedulable with the EDF algorithm if (Equation (11)):

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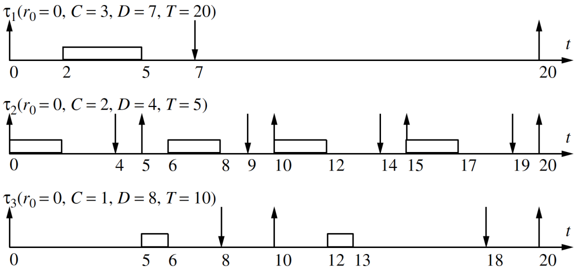
- At time $t = 0$, the three tasks are ready to execute and the task with the smallest absolute deadline is τ_2.



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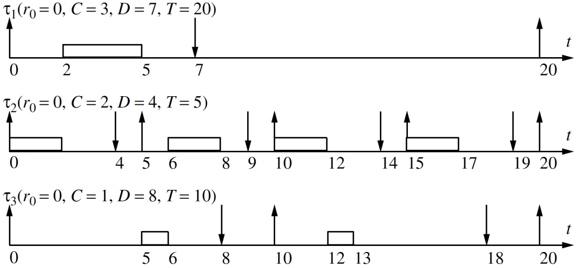

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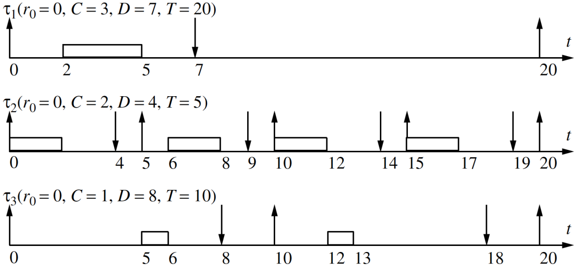

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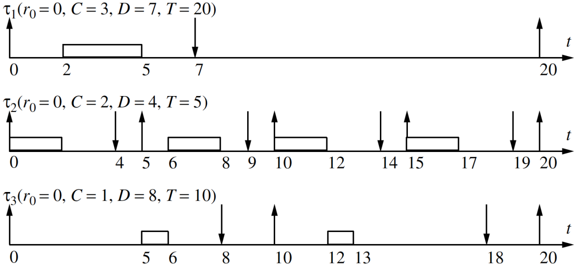

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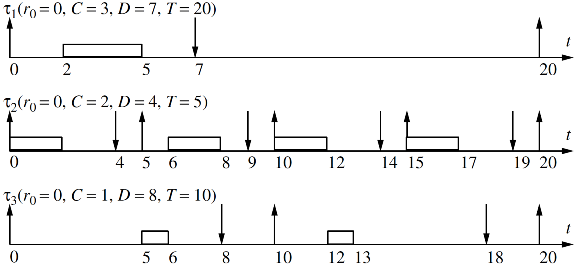
- At this point, task τ_2 is again ready. However, the task with the smallest absolute deadline is now τ_3, which begins to execute.



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- Please take a closer look at example Figure 2.9 on page 32 of the textbook

The end