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EPE 2165—Analog Electronics

SOLUTION #—1: DIODE CIRCUITS

July 26, 2022

4.1

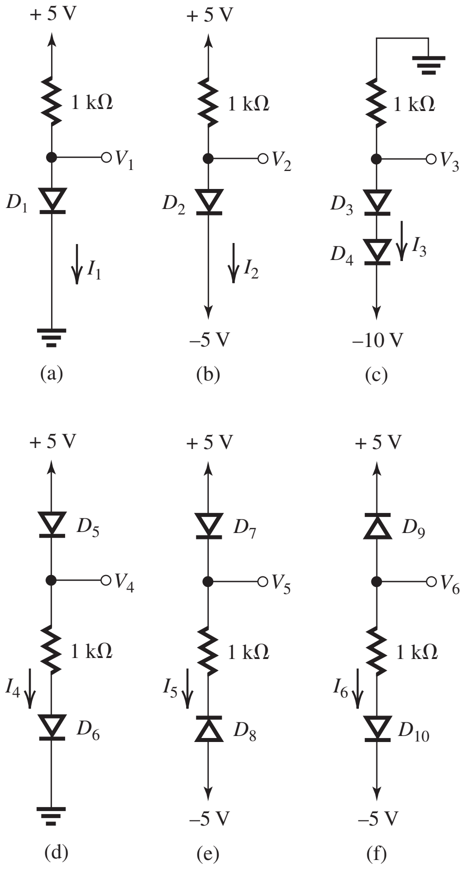

Figure 4.1.1

(a) Since the anode of D_1 is connected to $+5\,\text{V}$ through the 1-kΩ resistor, and the cathode is connected to ground, D_1 conducts and, being ideal, has a zero voltage drop, thus

$$V_1 = 0\,\text{V}$$
$$I_1 = \frac{5-0}{1\,\text{k}\Omega} = 5\,\text{mA}$$

(b) Here the situation is similar to that in (a) except that the cathode is connected to $-5\,\text{V}$. Thus, diode D_2 conducts and, being ideal, exhibits zero voltage drop, thus

$$V_2 = -5\,\text{V}$$
$$I_2 = \frac{5-(-5)}{1\,\text{k}\Omega} = 10\,\text{mA}$$

(c) Diodes D_3 and D_4 are connected in series in the same direction with the cathode of D_4 connected to $-10\,\text{V}$ and the anode of D_3 to ground through the 1-kΩ resistance. Thus, D_3 and D_4 will conduct and, being ideal, will have zero voltage drops, resulting in

$$V_3 = -10\,\text{V}$$
$$I_3 = \frac{0-(-10)}{1\,\text{k}\Omega} = 10\,\text{mA}$$

(d) Diodes D_5 and D_6 are connected in series in the same direction and since the anode of D_5 is connected to $+5\,\text{V}$ and the cathode of D_6 to ground, the diodes will conduct and exhibit zero voltage drops, resulting in

$$V_4 = +5\,\text{V}$$
$$I_4 = \frac{5}{1\,\text{k}\Omega} = 5\,\text{mA}$$

(e) Here diode D_7 is polarized to conduct while the series diode D_8 is not (its anode is at $-5\,\text{V}$ and its cathode is connected through the 1-kΩ resistor and D_7 to +5 V). Thus

$$I_5 = 0$$

However, since D_7 is polarized to conduct, it will conduct a minute current that flows through the meter connected to measure V_5. The conducting D_7 will exhibit a zero voltage drop and thus

$$V_5 = +5 - 0 = +5\text{ V}$$

(f) Here D_9 is polarized to not conduct, thus

$$I_6 = 0$$

Diode D_{10}, however, is connected to conduct and it will conduct a very small current that flows through the meter connected to measure V_6. The resulting voltage across D_{10} will be zero and the voltage across the 1-kΩ resistor will be negligibly small, thus

$$V_6 = -5\text{ V}$$

4.2

The circuit with all currents and voltages indicated is shown in Fig. 4.2.2 (refer Figure below).

Our analysis proceeded as follows: Diode D_1, with its anode connected to ground and its cathode connected to −8 V through R_1, must conduct. Thus, its cathode will be at 0 V, which enables us to determine the current through R_1 as

$$I_{R1} = \frac{0-(-8)}{R_1} = \frac{8}{1} = 8\text{ mA}$$

Next, we assume that D_2 will be conducting because its cathode is at 0 V and its anode is connected to +8 V through R_2. Thus, the current in R_2 can now be found as

$$I_{R2} = \frac{8-0}{2} = 4\text{ mA}$$

Now, since I_{R1} (8 mA) is greater than I_{R2} (4 mA), then D_1 and D_2 both conduct. Now, noting that succeeding resistances are progressively larger, assume that D_3 and D_4 also conduct, and then their anode and cathode voltages will all be zero volts. Thus

$$I_{R3} = \frac{0-(-8)}{4} = 2\text{ mA}$$

and

$$I_{R4} = \frac{8-0}{8} = 1\text{ mA}$$

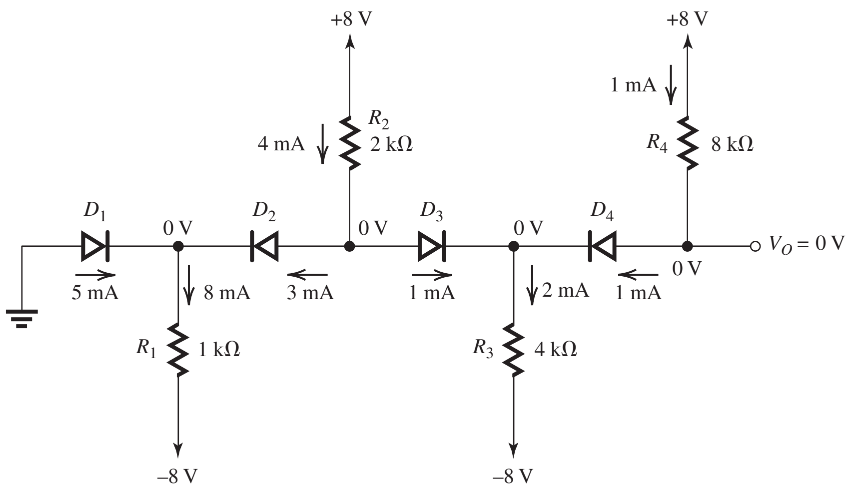

Figure 4.2.2

Thus, overall:

$$I_{D4} = I_{R4} = 1\text{ mA}$$
$$I_{D3} = I_{R3} - I_{D4} = 2 - 1 = 1\text{ mA}$$
$$I_{D2} = I_{R2} - I_{D3} = 4 - 1 = 3\text{ mA}$$
$$I_{D1} = I_{R1} - I_{D2} = 8 - 3 = 5\text{ mA}$$

and

$$V_O = 0\text{ V}$$

4.3

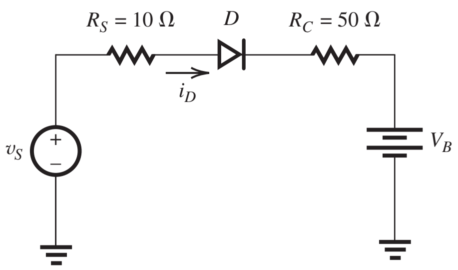

Figure 4.3.1

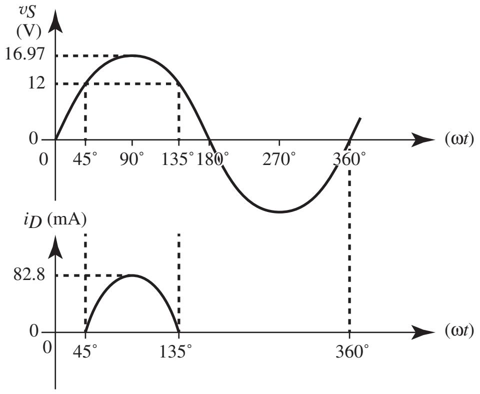

Figure 4.3.2

For a battery voltage V_B and an ideal diode, the diode current is

$$i_D = \frac{v_S - V_B}{R_S + R_C}$$
$$= \frac{\hat{V}_s \sin \omega t - V_B}{10 + 50}$$

where

$$\hat{V}_s = \sqrt{2} \times 12 = 16.97\text{ V}$$

Thus, for $V_B = 12$ V,

$$i_D = \frac{16.97 \sin \omega t - 12}{60\ \Omega}$$

which has a peak value of

$$\hat{i}_D = \frac{16.97 - 12}{60} = 82.8\text{ mA}$$

The diode begins to conduct (and ceases to conduct) when $16.97 \sin \omega t = 12$ V or $\sin \omega t = 12/16.97 = 0.707$, which correspond to an angle $\theta = \frac{\pi}{4} = 45°$. The average value of i_D can be computed as

$$i_D\Big|_{av} = \frac{1}{2\pi} \int_{\pi/4}^{3\pi/4} (16.97 \sin \theta - 12)\, d\theta / 60$$
$$= \frac{1}{2\pi \times 60} [-16.97 \cos \theta - 12\,\theta]_{\pi/4}^{3\pi/4}$$
$$= 13.7\text{ mA}$$

For $V_B = 14$ V,

$$\text{Peak current} = \frac{16.97 - 14}{60\ \Omega} = 49.5\text{ mA}$$

Conduction begins when $16.97 \sin \omega t = 14$, or $\sin \omega t = \frac{14}{16.97} = 0.825$, or $\omega t = 55.6°$. Thus,

Average current

$$= \frac{1}{2\pi} \int_{55.6°}^{180° - 55.6°} (16.97 \sin \theta - 14)\, d\theta / 60$$
$$= \frac{1}{2\pi \times 60} [-16.97 \cos \theta - 14\,\theta]_{55.6°}^{124.4°}$$
$$= 6.3\text{ mA}$$

4.4

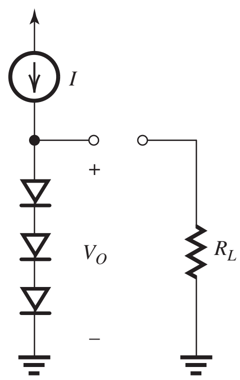

Figure 4.4.1

$$V_O = 3V_D$$

To obtain $V_O = 2.4$ V, we need to establish

$$V_D = 0.8\text{ V}$$

This is achieved when I is

$$I = I_S e^{V_D/V_T} = 10^{-16} e^{0.8/0.025} \simeq 7.9\text{ mA}$$

If R_L is connected and it draws a current of 1 mA, the current in the diodes will be reduced by 1 mA, to 6.9 mA, thus V_D becomes

$$V_D = V_T \ln(6.9 \times 10^{-3}/10^{-16}) = 0.797\text{ V}$$

and V_O becomes

$$V_O = 3 \times 0.797 = 2.39\text{ V}$$

Thus, V_O decreases by only 10 mV. Without R_L connected, since the diodes are fed with a constant current I, a change in temperature causes the voltage across each diode to change by -2 mV/°C. Thus, a temperature increase of 20°C causes V_D to decrease by $2 \times 20 = 40$ mV and causes V_O to decrease by 120 mV.

4.5

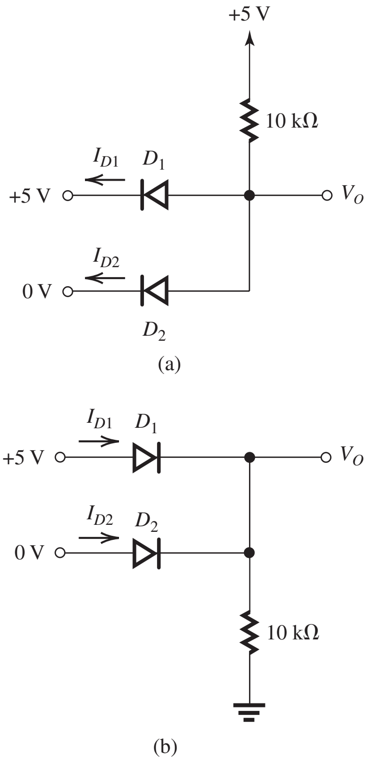

Figure 4.5.1

(a) For the circuit in Fig. 4.5.1(a), diode D_1 will be cut off while D_2 will conduct, causing the output voltage to become

$$V_O = 0 + V_{D2} = 0.7\text{ V}$$

Thus, D_1 has a 4.3-V reverse bias, which keeps it cut off. Thus,

$$I_{D1} = 0$$

and

$$I_{D2} = \frac{5 - 0.7}{10} = 0.43\text{ mA}$$

(b) For the circuit in Fig. 4.5.1(b), diode D_2 will be cut off while D_1 will conduct, causing V_O to become

$$V_O = 5 - V_{D1} = 5 - 0.7 = 4.3\text{ V}$$

Thus, D_2 will have a reverse-bias voltage of 4.3 V, which keeps it in the cut-off mode,

$$I_{D2} = 0$$

The current in D_1 can be found as

$$I_{D1} = \frac{V_O}{10\text{ k}\Omega} = \frac{4.3}{10} = 0.43\text{ mA}$$

4.6

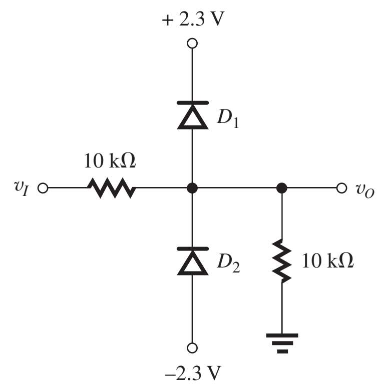

Figure 4.6.1

When v_O reaches $+3$ V, D_1 conducts and v_O remains at the $+3$-V level irrespective of the value of v_I. Thus, the upper limiting level is $+3$ V. v_O reaches $+3$ V when v_I is $+6$ V (because of the voltage divider comprised of the two 10-kΩ resistors), thus the upper input threshold voltage is $+6$ V.

A similar situation occurs in the negative direction: When v_O reaches -3 V, D_2 conducts and v_O remains at the -3-V level irrespective of the value of v_I. Thus, the lower limiting level is -3 V. v_O reaches -3 V when v_I reaches -6 V, thus the lower input threshold is -6 V.

In between the two threshold levels (i.e., for $-6\text{ V} \le v_I \le +6\text{ V}$), both D_1 and D_2 will be off and

$$v_O = \frac{10}{10+10} v_I = 0.5 v_I$$

Thus, the limiter gain K is

$$K = 0.5 \text{ V/V}$$

Figure 4.6.2 shows the transfer characteristic of the limiter.

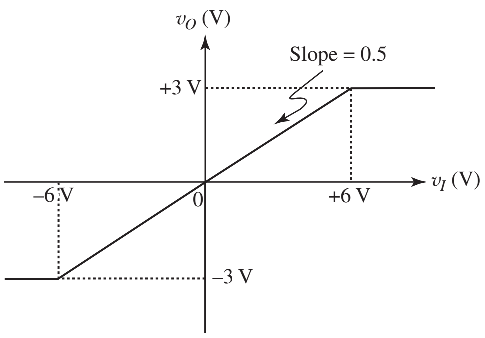

Figure 4.6.2

For $v_I = +12$ V, v_O will be $+3$ V, thus

$$i_I = \frac{12-3}{10\text{ k}\Omega} = 0.9 \text{ mA}$$

4.7

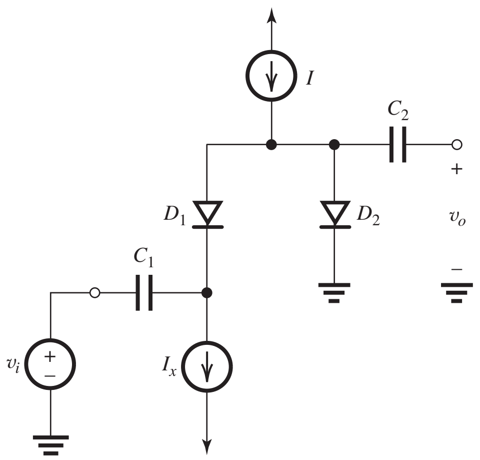

Figure 4.7.1

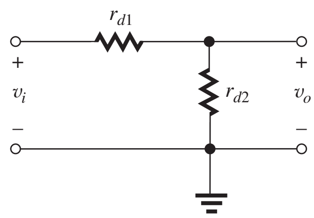

Figure 4.7.2

The dc currents in D_1 and D_2 are

$$I_{D1} = I_x$$

$$I_{D2} = I - I_{D1} = I - I_x$$

Thus, the small-signal resistances of D_1 and D_2 are

$$r_{d1} = \frac{V_T}{I_x}$$

$$r_{d2} = \frac{V_T}{I - I_x}$$

The small-signal equivalent circuit is shown in Fig. 4.7.2 from which we can find v_o/v_i as

$$\frac{v_o}{v_i} = \frac{r_{d2}}{r_{d2} + r_{d1}}$$

$$= \frac{V_T/(I-I_x)}{\frac{V_T}{I-I_x} + \frac{V_T}{I_x}}$$

Thus,

$$\frac{v_o}{v_i} = \frac{I_x}{I}$$

For $I = 1$ mA,

$$\frac{v_o}{v_i} = I_x$$

where I_x is in mA.

To obtain the given values of v_o/v_i, we need the corresponding values of I_x as follows:

v_o/v_i	0	0.2	0.5	1.0
I_x (mA)	0	0.2	0.5	1.0

This circuit is a signal attenuator whose transfer ratio is linearly controlled by the current I_x.

4.8

Figure 4.8.1 shows the circuit.

$$V_O = 4V_D$$

$$\Rightarrow V_D = \frac{3.0}{4} = 0.75 \text{ V}$$

Now, for the available diode, we have

$$0.1 \text{ mA} = I_S e^{0.7/0.025} \quad (1)$$

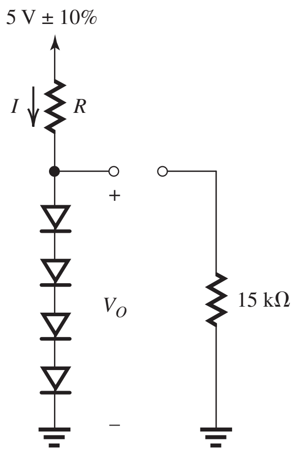

Figure 4.8.1

Thus, the required current I can be found from

$$I = I_S e^{0.75/0.025} \quad (2)$$

Dividing Eq. (2) by Eq. (1) yields

$$I = 0.1 \times e^{0.05/0.025} = 0.74 \text{ mA}$$

The required value of R can now be found from

$$I = \frac{5 - V_O}{R}$$

$$0.74 = \frac{5 - 3}{R}$$

$$\Rightarrow R = 2.7 \text{ k}\Omega$$

At a current $I_D = 0.74$ mA, each diode has a small-signal resistance of

$$r_d = \frac{V_T}{I_D} = \frac{25 \text{ mV}}{0.74 \text{ mA}} = 33.8 \ \Omega$$

and the string of four diodes has an incremental (small-signal) resistance of

$$r = 4r_d = 4 \times 33.8 = 135 \ \Omega$$

With the supply voltage changing by $\pm 10\%$—that is, by ± 0.5 V—we can use the voltage divider comprised of the diode string (with a total resistance $r = 135 \ \Omega$) and the resistance R to determine the corresponding change in V_O as

$$\triangle V_O = \pm 0.5 \times \frac{0.135}{0.135 + 2.7}$$

$$= \pm 23.8 \text{ mV}$$

or $\pm 0.8\%$.

This implies a change of $\pm\frac{23.8}{4} \simeq \pm 6$ mV across each diode, which is small enough for the small-signal model to be applicable.

If a load resistance $R_L = 15$ kΩ is connected to V_O, we can determine the approximate change $\triangle V_O$ in V_O as follows. First, assume that V_O does not change, then the current drawn by R_L can be found as

$$I_L = \frac{V_O}{R_L} = \frac{3}{15} = 0.2 \text{ mA}$$

This current will be subtracted from that supplied through R, thus the current in the diode string is reduced by 0.2 mA. Using the small signal model for the diodes, each diode voltage will decrease by $0.2 \text{ mA} \times r_d = 0.2 \times 33.8 = 6.76$ mV, which is still small enough for the small signal model to be applicable. Thus, the change in V_O will be

$$\triangle V_O = -4 \times 6.76 = -27 \text{ mV}$$

or $= 0.9\%$.

Since the change in V_O is small, our original assumption used to find the current I_L is justified.

4.9

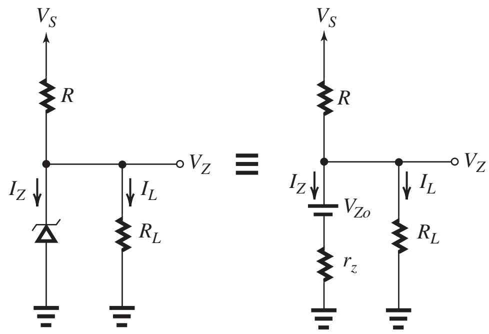

Figure 4.9.1

$$V_{ZK} \simeq V_{Z0}$$

where

$$V_{Z0} + r_z I_Z = V_Z$$

Thus,

$$V_{Z0} = V_Z - r_z I_Z$$

$$= 6.8 - 0.020 \times 5$$

$$= 6.7 \text{ V}$$

For no load, the lowest value of V_S to maintain breakdown operation is equal to the knee voltage, that is,

$$V_{S\min} = 6.7 \text{ V}$$

For $V_S = 9$ V, the maximum load current $I_{L\max}$ for which breakdown operation is maintained is found from

$$V_S = V_{Z0} + (I_{L\max} + I_{ZK})R$$

where we have assumed the zener voltage to be at its knee value ($V_{ZK} \simeq V_{Z0}$) and the zener current at $I_{ZK} = 0.2$ mA. Thus

$$9 = 6.7 + (I_{L\max} + 0.2) \times 0.2$$
$$\Rightarrow I_{L\max} = 11.3 \text{ mA}$$

For $I_L = 0.5\, I_{L\max} = 5.65$ mA, the lowest value of V_S to maintain breakdown operation is found from

$$V_{S\min} = V_{Z0} + (0.5\, I_{L\max} + I_{ZK})R$$
$$= 6.7 + (0.5 \times 11.3 + 0.2) \times 0.2$$
$$= 7.9 \text{ V}$$

$$\text{Line regulation} = \frac{r_z}{r_z + R} = \frac{20}{20 + 200} \simeq 91 \text{ mV/V}$$

$$\text{Load regulation} = -(r_z \| R) = -(20\|200)$$
$$= -18.2 \text{ mV/mA}$$

4.10 Refer to Figures 4.10.1 and 4.10.2 below.

The circuit in Fig. 4.10.1 is a full-wave rectifier with a center-tapped secondary winding. The circuit can be analyzed by looking at v_O^+ and v_O^- separately. The circuits for doing so are shown in Fig. 4.10.2. Their analysis, of course, is identical. For each supply,

$$V_O = 10 \text{ V}$$
$$V_r = 0.8 \text{ V}$$

Thus,

$$v_O = 10 \pm 0.4 \text{ V}$$

It follows that the peak value of v_S must be $10.4 + 0.7 = 11.1$ V and the total rms voltage across the secondary will be

$$\frac{2 \times 11.1}{\sqrt{2}} = 15.7 \text{ V (rms)}$$

Thus,

$$\text{Transformer turns ratio} = \frac{120}{15.7} = 7.64 : 1$$

To deliver a 50-mA dc current to each load,

$$R = \frac{10}{0.05} = 200\ \Omega$$

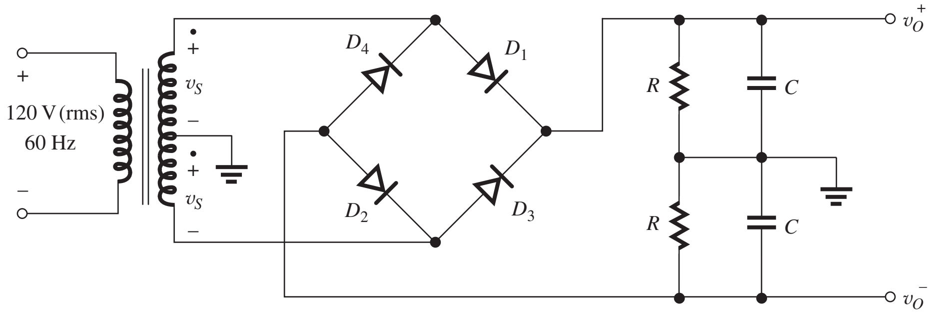

Figure 4.10.1

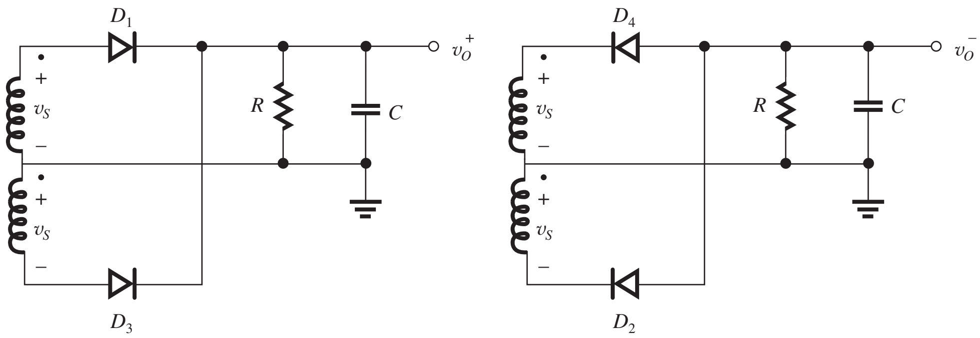

Figure 4.10.2

Now, the value of C can be found from

$$V_r = \frac{V_p - 0.7}{2fCR}$$
$$0.8 = \frac{11.1 - 0.7}{2 \times 60 \times C \times 200}$$
$$\Rightarrow C = 541.7\ \mu\text{F}$$

To specify the diodes, we determine i_{Dav} and $i_{D\max}$,

$$i_{Dav} = I_L\left[1 + \pi\sqrt{(V_p - 0.7)/2V_r}\right]$$
$$= 50\left[1 + \pi\sqrt{(11.1 - 0.7)/1.6}\right]$$
$$= 450.5\text{ mA}$$
$$i_{D\max} = I_L\left[1 + 2\pi\sqrt{(V_p - 0.7)/2V_r}\right]$$
$$= 50\left[1 + 2\pi\sqrt{(11.1 - 0.7)/1.6}\right]$$
$$= 851\text{mA}$$

To determine the required PIV rating of each diode, we determine the maximum reverse voltage that appears across one of the diodes, say D_1. This occurs when v_S is at its maximum negative value $-V_p$. Since the cathode of D_1 will be at $+10.4$ V, the maximum reverse voltage across D_1 will be $10.4 + 11.1 = 21.5$ V. Using a factor of safety of 1.5, then each of the four diodes must have

$$\text{PIV} = 1.5 \times 21.5 = 32.3\text{ V}$$

4.11

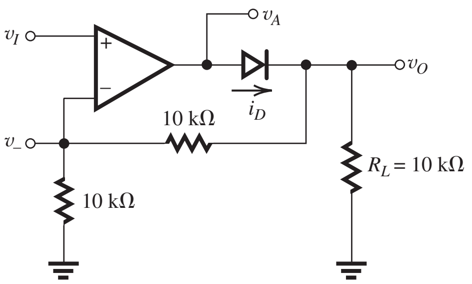

Figure 4.11.1

When v_I is positive, v_A goes positive, turning on the diode and closing the negative feedback loop around the op amp. The result is that $v_- = v_I$, $v_O = 2\,v_- = 2\,v_I$, $v_A = v_O + 0.7$, and $i_D = \frac{v_O}{20} + \frac{v_O}{10}$ $= 0.15\,v_O$, mA. Thus,

(a) For $v_I = +0.1$ V : $v_- = +0.1$ V, $v_O = +0.2$ V, $v_A = 0.9$ V, $i_D = 0.03$ mA $= 30\ \mu$A.

(b) For $v_I = +1$ V : $v_- = +1$ V, $v_O = +2$ V, $v_A = +2.7$ V, $i_D = 0.15 \times 2 = 0.3$ mA.

When v_I goes negative, v_A follows, the diode turns off, and the feedback loop is opened. The op amp saturates with $v_A = -5$ V, $v_- = 0$ V, and $v_O = 0$ V. Thus,

(c) For $v_I = -0.1$ V : $v_- = 0$ V, $v_A = -5$ V, $v_O = 0$ V, and $i_D = 0$.

(d) For $v_I = -1$ V : $v_- = 0$ V, $v_A = -5$ V, $v_O = 0$ V, and $i_D = 0$.

Finally, if v_I is a symmetrical square wave of 2-V amplitude and zero average, the output will be zero during the negative half-cycles of the input and will equal twice the input (i.e., 4 V) during the positive half-cycles. See Fig. 4.11.2.

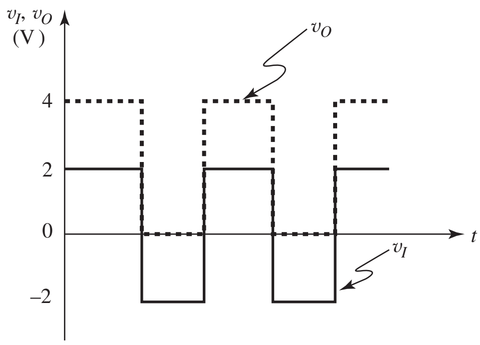

Figure 4.11.2

Thus, v_O is a square wave with 0-V and 4-V levels (i.e., 2-V average) and, of course, the same frequency (1 kHz) as the input.

4.12

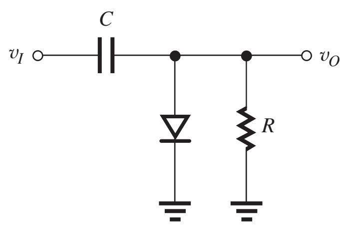

Figure 4.12.1

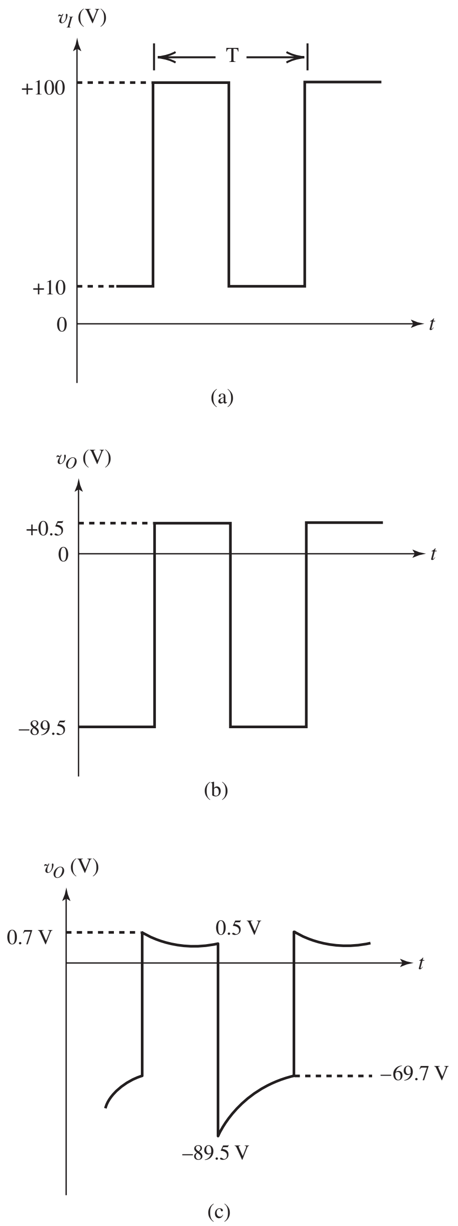

Figure 4.12.2

(a) For light load (R very large), the output is a square wave of period T going from $+0.5$ V to $+0.5 - (100 - 10) = -89.5$ V. See Figure 4.12.2(b).

(b) If R is reduced so that $RC = 2T$, the output waveform will take the shape shown in Fig. 4.12.2(c). Consider first the negative portion of the waveform. As before, the largest negative value is -89.5 V. At this time, the diode is cut off and the capacitor discharges through the now smaller resistance R. Thus, v_O rises exponentially, heading towards 0 V, but of course is interrupted $(T/2)$ second later, thus

$$v_O(t) = 0 - 89.5e^{-t/CR}$$

$$v_O\left(\frac{T}{2}\right) = -89.5e^{-T/2CR}$$

For $CR = 2T$,

$$v_O\left(\frac{T}{2}\right) = -89.5 \times e^{-1/4} = -69.7 \text{ V}$$

Thus, at the end of the discharge interval, $v_O = -69.7$ V. At that time, the input rises by 90 V and the output attempts to follow. However, the diode clamps v_O to about 0.7 V or so and it conducts heavily to recharge the capacitor. The diode voltage then drops, reaching about 0.5 V at the end of the half-cycle. As the input falls by 90 V, the output follows, reaching -89.5 V, and the cycle repeats.