UNIVERSITY OF RWANDA

COLLEGE OF SCIENCE & TECHNOLOGY

SCHOOL OF ENGEERING

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

KIZITO NKURIKIYEYEZU, PHD

EPE 2165—ANALOG ELECTRONICS

SOLUTION #—3: DIODE CIRCUITS

July 26, 2022

6.1

(a)

$$i_C = I_S e^{v_{BE}/V_T}$$

At $i_C = i_{C1}$,

$$i_{C1} = I_S e^{v_{BE1}/V_T} \tag{1}$$

and at $i_C = i_{C2}$,

$$i_{C2} = I_S e^{v_{BE2}/V_T} \tag{2}$$

Dividing Eq. (2) by Eq. (1), we obtain

$$\frac{i_{C2}}{i_{C1}} = e^{(v_{BE2}-v_{BE1})/V_T} \tag{3}$$

Which can be expressed alternately as

$$v_{BE2} - v_{BE1} = V_T \ln\left(\frac{i_{C2}}{i_{C1}}\right) \tag{4}$$

For $i_{C2}/i_{C1} = 2$,

$$\triangle v_{BE} \equiv v_{BE2} - v_{BE1} = 25 \ln 2 = 17.3 \text{ mV}$$

(b) Using Eq. (4) with $i_{C2}/i_{C1} = 10$, we find

$$\triangle v_{BE} \equiv v_{BE2} - v_{BE1} = 25 \ln(10) = 57.6 \text{ mV}$$

(c) The percentage change in i_C,

$$\frac{i_{C2} - i_{C1}}{i_{C1}} \times 100$$

corresponding to a change $\triangle v_{BE} \equiv v_{BE2} - v_{BE1}$ can be calculated using Eq. (3). The results obtained for the given values of $\triangle v_{BE}$ are as in the table below:

We observe that for small $\triangle v_{BE}$, positive and negative increments yield nearly equal positive and negative changes in i_C. Also, for small $\triangle v_{BE}$, the changes in i_C are linearly related to $\triangle v_{BE}$ with a proportionality factor of 4% per mV. This linear relationship breaks down when $\triangle v_{BE}$ becomes large, for instance, ± 10 mV.

(d)

$$\beta = \frac{i_C}{i_B} = \frac{1 \text{ mA}}{12.5\ \mu\text{A}} = \frac{1}{0.0125} = 80$$

$$\alpha = \frac{\beta}{\beta+1} = \frac{80}{81} = 0.988$$

$$i_E = i_C + i_B$$

$$= 1 + 0.0125 = 1.0125 \text{ mA}$$

(e)

$$1 \times 10^{-3} = I_S e^{675/25}$$

$$\Rightarrow I_S = 1.88 \times 10^{-15} \text{ A}$$

(f) With two identical transistors connected in parallel with a combined collector current of 1 mA, each transistor has a collector current of 0.5 mA. Thus, Eq. (4) can be used to determine the change in v_{BE} as i_C decreases from 1 mA to 0.5 mA as

$$v_{BE2} - v_{BE1} = 25 \ln(0.5) = -17.3 \text{ mV}$$

From (e) above, $v_{BE1} = 675$ mV, thus

$$v_{BE2} = 675 - 17.3 = 657.7 \text{ mV}$$

$\triangle v_{BE}$, mV	+0.5	−0.5	+1.0	−1.0	+2.0	−2.0	+5.0	−5.0	+10	−10
$\frac{\triangle i_C}{i_C}$, %	+2	−2	−4	−4	+8.3	−7.7	+22	−18	+49	−33

Alternatively, we can think of the combination of the two parallel devices as a transistor with twice the base-emitter area and correspondingly twice the value of I_S. When this equivalent transistor is conducting a 1-mA collector current, the base emitter voltage can be obtained from

$$1 \times 10^{-3} = 2 \times 1.88 \times 10^{-15}\, e^{v_{BE}/V_T}$$

$$\Rightarrow v_{BE} = 657.7\text{ mV}$$

6.2

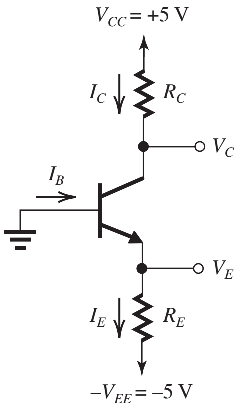

Figure 6.2.1

(a) $V_C = +1.58$ V and $V_B = 0$ V; thus, $V_C > V_B$, indicating that the transistor must be operating in the active mode.

(b)

$$I_E = \frac{V_E - (-V_{EE})}{R_E} = \frac{-0.68 - (-5)}{5}$$

$$I_E = 0.864\text{ mA}$$

$$I_B = \frac{I_E}{\beta + 1} = \frac{0.864}{100 + 1} = 0.0086\text{ mA}$$

$$I_C = I_E - I_B = 0.855\text{ mA}$$

(c)

$$I_C = \frac{V_{CC} - V_C}{R_C}$$

$$0.855 = \frac{5 - 1.58}{R_C}$$

$$\Rightarrow R_C = 4\text{ k}\Omega$$

(d) To obtain $I_C = 2$ mA, the emitter current I_E must be

$$I_E = \frac{I_C}{\alpha}$$

where

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

Thus,

$$I_E = \frac{2}{0.99} = 2.02\text{ mA}$$

Next, we determine V_E by first finding V_{BE}. The transistor has $I_C = 0.855$ mA at $V_{BE} = 0.68$ V. To obtain $I_C = 2$ mA, V_{BE} must be increased by

$$\triangle V_{BE} = V_T \ln \frac{2}{0.855} = 0.02\text{ V}$$

resulting in

$$V_{BE} = 0.68 + 0.02 = 0.7\text{ V}$$

and thus

$$V_E = 0 - V_{BE} = -0.7\text{ V}$$

The required value of R_E can now be determined from

$$R_E = \frac{V_E - (-V_{EE})}{I_E} = \frac{-0.7 - (-5)}{2.02}$$

$$\Rightarrow R_E = 2.13\text{ k}\Omega$$

The required value of R_C can be determined from

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{5 - 1}{2}$$

$$\Rightarrow R_C = 2\text{ k}\Omega$$

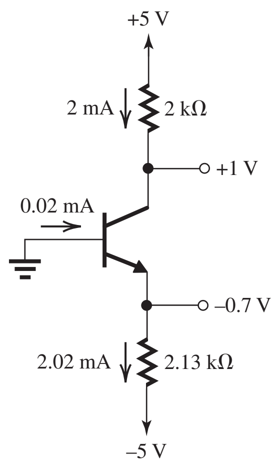

Figure 6.2.2

Figure 6.2.2 shows the redesigned circuit together with all voltages and currents.

6.3

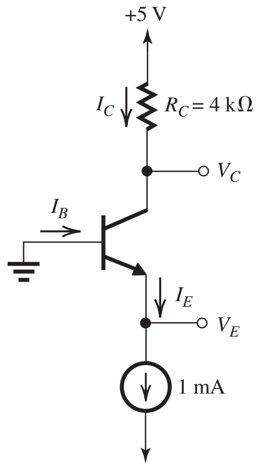

Figure 6.3.1

Since $I_E = 1$ mA, it remains constant and independent of the value of β.

$$\alpha = \frac{\beta}{\beta + 1}$$

$$\beta = 50, \qquad \alpha = \frac{50}{50+1} = 0.980$$

$$\beta = 200, \qquad \alpha = \frac{200}{200+1} = 0.995$$

Thus, α lies in the range 0.980 to 0.995. Assuming the transistor is operating in the active mode over this range, we find I_C from

$$I_C = \alpha I_E$$

For $I_E = 1$ mA, I_C lies in the range 0.980 mA to 0.995 mA.

$$V_C = V_{CC} - I_C R_C = 5 - I_C \times 4$$

Substituting for $I_C = 0.980$ to 0.995 results in the range of V_C as 1.08 V to 1.02 V. Note that over this range of V_C, the BJT is indeed in the active region, as assumed.

$$I_B = \frac{I_E}{\beta+1} = \frac{1}{\beta+1} \text{ mA}$$

For β in the range 50 to 200, I_B will be in the range 0.02 mA to 0.005 mA. Utilizing the relationship

$$I_C = I_S e^{V_{BE}/V_T}$$

which can be rewritten as

$$V_{BE} = V_T \ln(I_C/I_S)$$

we can determine the range of V_{BE} by substituting $I_C = 0.980$ mA to 0.995 mA. We obtain the range of V_{BE} as from 0.690 V to 0.691 V. Since $V_E = -V_{BE}$, the range of V_E will be -0.690 V to -0.691 V.

Comment:
Although β ranges over a 4:1 range or a 300% change relative to its low value, the corresponding changes in α and all voltages and currents (except for I_B) are much lower. For instance, α, I_C, and V_C change by only 1.5% relative to their low values. V_{BE} and V_E change by only 1 mV (in 700 mV or so; a negligible change). The only quantity that tracks the change in β is I_B. Nevertheless, as we will learn later on, I_B in this circuit is not an important parameter. The insensitivity of this circuit to β variation makes it an excellent design!

6.4

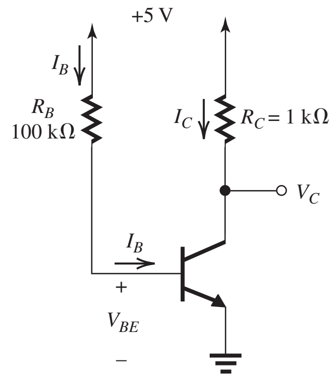

Figure 6.4.1

The current I_B can be determined from

$$I_B = \frac{5 - V_{BE}}{R_B} = \frac{5 - 0.7}{100}$$

Thus,

$$I_B = 0.043 \text{ mA}$$

and is independent of β.

Assuming active-mode operation, the collector current can be found from

$$I_C = \beta I_B$$

and the collector voltage can then be determined using

$$V_C = 5 - R_C I_C = 5 - 1 \times I_C$$

For $\beta = 50$,

$$I_C = 50 \times 0.043 = 2.15 \text{ mA}$$

and

$$V_C = 5 - 1 \times 2.15 = 2.85 \text{ V}$$

which is greater than the voltage at the base, thus the transistor is the active region, as assumed.

For $\beta = 200$,

$$I_C = 200 \times 0.043 = 8.6 \text{ mA}$$

and

$$V_C = 5 - 1 \times 8.6 = -3.6 \text{ V}$$

which is impossible as the base voltage is $+0.7$ V. Thus, the transistor cannot be operating in the active mode. Rather, it must be in the saturation mode, for which

$$V_C = V_{CEsat} = 0.2 \text{ V}$$
$$I_C = \frac{5 - V_C}{R_C} = \frac{5 - 0.2}{1} = 4.8 \text{ mA}$$

The ratio I_C/I_B, which is the forced β, is thus

$$\beta_{\text{forced}} = \frac{I_C}{I_B} = \frac{4.8}{0.043} = 111.6$$

which is lower than the normal value of β (200), confirming that the transistor is in saturation.

Comment:
The operation of this circuit is highly sensitive to the value of β. Indeed, over the specified range of β, the transistor goes from active mode to saturation. This is not a desirable situation and the circuit is not a good design.

6.5

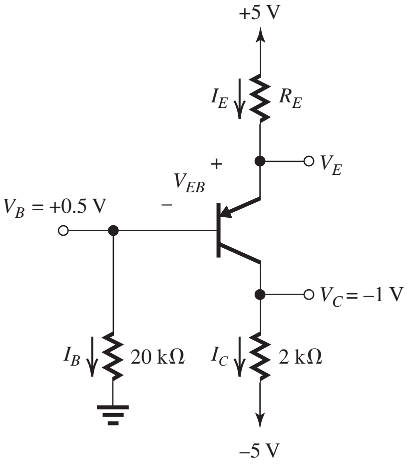

Figure 6.5.2

From Fig. 6.5.2, we see that V_C is lower than V_B, thus the *pnp* transistor is operating in the active mode. By reference to the figure, we can write

$$I_B = \frac{V_B}{20\text{ k}\Omega} = \frac{+0.5\text{ V}}{20\text{ k}\Omega} = 0.025 \text{ mA}$$

and

$$I_C = \frac{V_C - (-5)}{2\text{ k}\Omega} = \frac{-1+5}{2} = 2 \text{ mA}$$

Thus,

$$\beta \equiv \frac{I_C}{I_B} = \frac{2\text{ mA}}{0.025\text{ mA}} = 80$$

To obtain V_{EB}, we utilize the given information that $V_{BE} = 0.7$ V at $I_C = 1$ mA. Here $I_C = 2$ mA, thus

$$V_{BE} = 0.7 + V_T \ln\left(\frac{2}{1}\right) = 0.717 \text{ V}$$

We now can find V_E as

$$V_E = V_B + V_{EB}$$
$$= 0.5 + 0.717 = 1.217 \text{ V}$$

The current I_E can be found as

$$I_E = I_C + I_B = 2 + 0.025 = 2.025 \text{ mA}$$

The value of R_E can be determined from

$$I_E = \frac{5 - V_E}{R_E}$$
$$\Rightarrow R_E = \frac{5 - 1.217}{2.025} = 1.868 \text{ k}\Omega$$

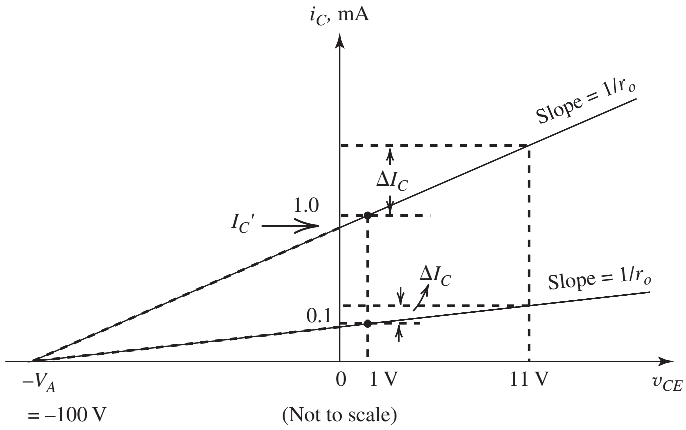

Figure 6.6.1

6.6

(a) Refer to Figure 6.6.1 above.

$$r_o = \frac{V_A}{I_C} \simeq \frac{100\text{ V}}{1\text{ mA}} = 100\text{ k}\Omega$$

Note that the approximation involved is that we used I_C at $V_{CE} = 1$ V rather than I'_C, which would be the value at the intersection of the $i_C - v_{CE}$ line with the vertical axis (Fig. 6.6.1). Alternatively, we can use

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{100+1}{1} = 101\text{ k}\Omega$$

which is very close to the approximate value obtained above and which we will usually use.

(b) For $\triangle V_{CE} = 10$ V, the current I_C changes by

$$\triangle I_C = \frac{\triangle V_{CE}}{r_o} = \frac{10}{100} = 0.1\text{ mA}$$

Thus, I_C becomes

$$I_C = 1 + 0.1 = 1.1\text{ mA}$$

(c)

$$r_o = \frac{V_A}{I_C} \simeq \frac{100\text{ V}}{0.1\text{ mA}} = 1000\text{ k}\Omega = 1\text{ M}\Omega$$

$$\triangle I_C = \frac{\triangle V_{CE}}{r_o} = \frac{10\text{ V}}{1000\text{ k}\Omega} = 0.01\text{ mA}$$

$$I_C = 0.1 + 0.01 = 0.11\text{ mA}$$

6.7

(a)

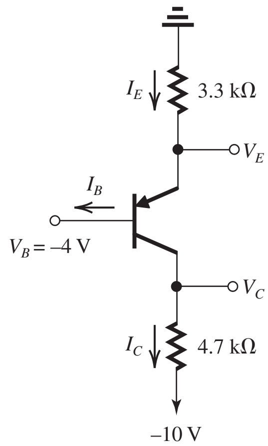

Figure 6.7.1(a)

Assume operation in the active mode.

$$V_E = V_B + V_{EB} = -4 + 0.7 = -3.3\text{ V}$$

$$I_E = \frac{0 - V_E}{3.3\text{ k}\Omega} = \frac{0-(-3.3)}{3.3} = 1\text{ mA}$$

$$I_C = \alpha I_E = \frac{\beta}{\beta+1} I_E = \frac{50}{50+1} \times 1 = 0.98\text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.98}{50} = 0.0196\text{ mA} = 19.6\ \mu\text{A}$$

$$V_C = -10 + I_C \times 4.7 = -10 + 0.98 \times 4.7 = -5.39\text{ V}$$

Since $V_C < V_B$, the CBJ is reverse biased and the *pnp* transistor is operating in the active mode, as assumed.

(b)

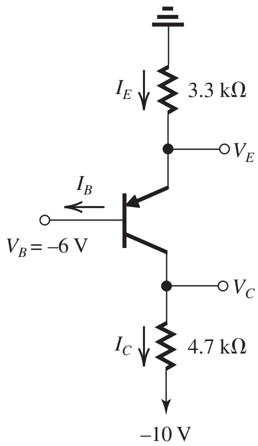

Figure 6.7.1(b)

Assume active-mode operation.

$$V_E = V_B + V_{EB} = -6 + 0.7 = -5.3 \text{ V}$$
$$I_E = \frac{0 - V_E}{3.3 \text{ k}\Omega} = \frac{0 - (-5.3)}{3.3} = 1.606 \text{ mA}$$
$$I_C = \alpha I_E = \frac{\beta}{\beta + 1} I_E = \frac{50}{50 + 1} \times 1.606$$
$$= 1.57 \text{ mA}$$
$$V_C = -10 + 4.7 \times I_C = -10 + 4.7 \times 1.57$$
$$= -2.62 \text{ V}$$

Since V_C at -2.62 V is higher than V_B at -6 V, it follows that the transistor is not in the active mode as we assumed. Rather, the *pnp* transistor must be operating in saturation. In this case, V_E and I_E remain unchanged at

$$V_E = -5.3 \text{ V}, \qquad I_E = 1.606 \text{ mA}$$

but V_{EC} now is

$$V_{EC\text{sat}} = 0.2 \text{ V}$$

Thus,

$$V_C = V_E - V_{EC\text{sat}} = -5.3 - 0.2 = -5.5 \text{ V}$$

and

$$I_C = \frac{V_C - (-10)}{4.7 \text{ k}\Omega} = \frac{-5.5 + 10}{4.7} = 0.957 \text{ mA}$$

and

$$I_B = I_E - I_C = 1.606 - 0.957 = 0.649 \text{ mA}$$

As another check that the transistor is operating in saturation, we find the forced β as

$$\beta_{\text{forced}} \equiv \frac{I_C}{I_B} = \frac{0.957 \text{ mA}}{0.649 \text{ mA}} = 1.47$$

which is much lower than the normal β of 50, verifying that the transistor is operating in saturation.

(c)

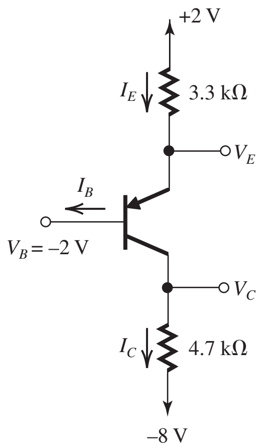

Figure 6.7.1(c)

Assume active-mode operation.

$$V_E = V_B + V_{EB} = -2 + 0.7 = -1.3 \text{ V}$$
$$I_E = \frac{+2 - V_E}{3.3 \text{ k}\Omega} = \frac{2 - (-1.3)}{3.3} = 1 \text{ mA}$$
$$I_C = \alpha I_E = \frac{\beta}{\beta + 1} I_E = \frac{50}{50 + 1} \times 1 = 0.98 \text{ mA}$$
$$I_B = \frac{I_C}{\beta} = \frac{0.98 \text{ mA}}{50} = 0.0196 \text{ mA} = 19.6\ \mu\text{A}$$
$$V_C = -8 + I_C \times 4.7 = -8 + 0.98 \times 4.7 = -3.39 \text{ V}$$

Since V_C at -3.39 V is lower than V_B at -2 V, the CBJ is reverse biased and the transistor is operating in the active mode, as assumed.

(d)

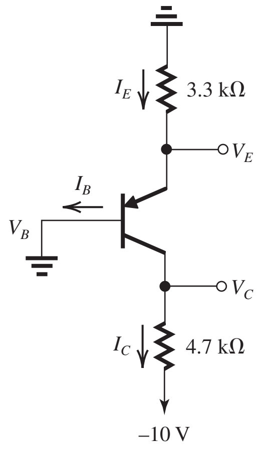

Figure 6.7.1(d)

Since the base is at 0 V and the emitter is connected to ground (0 V) through the 3.3-kΩ resistance, the emitter-base junction cannot conduct. Thus,

$$V_E = 0\text{ V}$$

$$I_E = 0\text{ mA}$$

Since the collector is connected to -10 V through the 4.7-kΩ resistance, the CBJ will be reverse biased. Thus,

$$I_C = 0\text{ mA}$$

$$I_B = 0$$

and

$$V_C = -10 + I_C \times 4.7 = -10\text{ V}$$

Thus, the transistor is cut off.

(e)

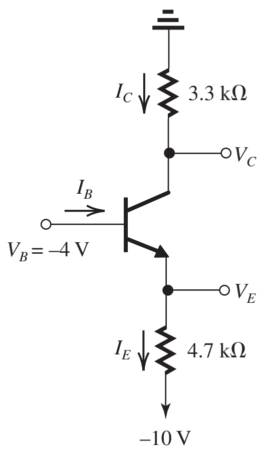

Figure 6.7.1(e)

Assume active-mode operation.

$$V_E = V_B - V_{BE} = -4 - 0.7 = -4.7\text{ V}$$

$$I_E = \frac{V_E - (-10)}{4.7\text{ k}\Omega} = \frac{-4.7 + 10}{4.7} = 1.128\text{ mA}$$

$$I_C = \alpha\, I_E = \frac{\beta}{\beta + 1} I_E = \frac{50}{50 + 1} \times 1.13 = 1.105\text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1.105}{50} = 0.022\text{ mA} = 22\ \mu\text{A}$$

$$V_C = 0 - I_C \times 3.3 = -1.105 \times 3.3 = -3.65\text{ V}$$

Since V_C at -3.65 V is higher than V_B at -4 V, the CBJ is reverse biased and the *npn* transistor is operating in the active mode, as assumed.

(f)

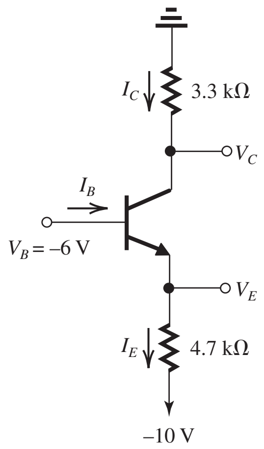

Figure 6.7.1(f)

Assume active-mode operation.

$$V_E = V_B - V_{BE} = -6 - 0.7 = -6.7\text{ V}$$

$$I_E = \frac{V_E - (-10)}{4.7\text{ k}\Omega} = \frac{-6.7 + 10}{4.7} = 0.702\text{ mA}$$

$$I_C = \alpha I_E = \frac{\beta}{\beta + 1} I_E = \frac{50}{50 + 1} \times 0.702 = 0.688\text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.688}{50} = 0.0138\text{ mA} = 13.8\ \mu\text{A}$$

$$V_C = 0 - I_C \times 3.3 = 0 - 0.688 \times 3.3 = -2.27\text{ V}$$

Since V_C at -2.27 V is higher than V_B at -6 V, the CBJ is reverse biased and the *npn* transistor is operating in the active mode, as assumed.

6.8

(a)

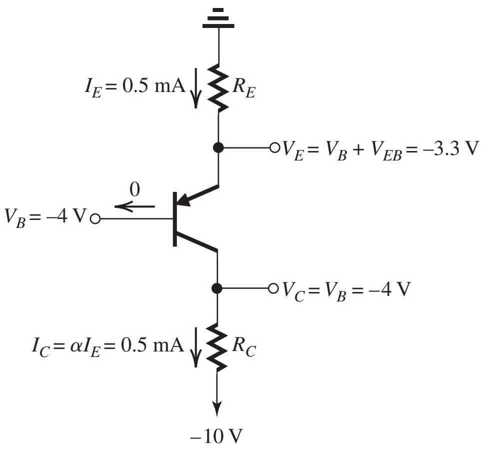

Figure 6.8.1

We note that $V_{BC} = 0$ means the transistor is operating in the active mode. The circuit is shown in Fig. 6.8.1 with some of the analysis already done on the diagram. We can now write

$$R_E = \frac{0 - V_E}{I_E} = \frac{0 - (-3.3)}{0.5\text{ mA}} = 6.6\text{ k}\Omega$$

$$R_C = \frac{V_C - (-10)}{I_C} = \frac{-4 + 10}{0.5\text{ mA}} = 12\text{ k}\Omega$$

(b)

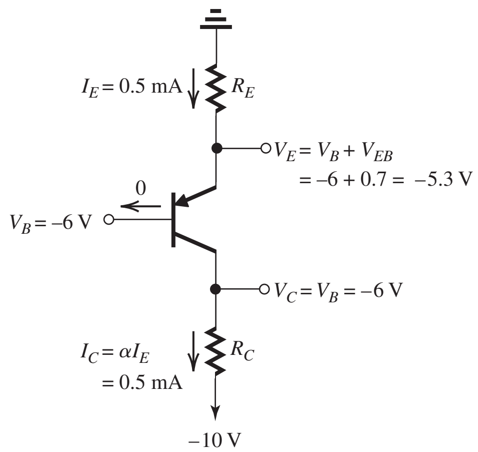

Figure 6.8.2

The circuit with some of the analysis already performed directly on the diagram is shown in Fig. 6.8.2. We can now write

$$R_E = \frac{0 - V_E}{I_E} = \frac{0 - (-5.3)}{0.5} = 10.6\text{ k}\Omega$$

$$R_C = \frac{V_C - (-10)}{I_C} = \frac{-6 + 10}{0.5} = 8\text{ k}\Omega$$

6.9

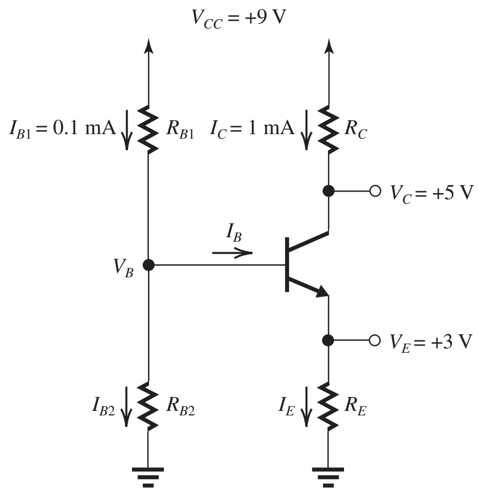

Figure 6.9.2

The circuit is shown in Fig. 6.9.2 with the required current and voltage values indicated. Observe that since V_C is higher than V_E by more than 0.3 V, the BJT will be operating in the active mode. The required value of R_C can be found from

$$R_C = \frac{V_{CC} - V_C}{I_C} = \frac{9 - 5}{1\text{ mA}} = 4\text{ k}\Omega$$

We can determine I_B from

$$I_B = \frac{I_C}{\beta} = \frac{1\text{ mA}}{100} = 0.01\text{ mA}$$

and thus

$$I_E = I_C + I_B = 1 + 0.01 = 1.01\text{ mA}$$

Now, the value of R_E can be found from

$$R_E = \frac{V_E}{I_E} = \frac{3\text{ V}}{1.01\text{ mA}} = 2.97\text{ k}\Omega$$

The base voltage V_B can be found as

$$V_B = V_E + V_{BE} = 3 + 0.7 = 3.7\text{ V}$$

The value of R_{B1} can be then found from

$$R_{B1} = \frac{V_{CC} - V_B}{I_{B1}} = \frac{9 - 3.7}{0.1 \text{ mA}} = 53 \text{ k}\Omega$$

A node equation at the base yields the value of the current I_{B2} as

$$\begin{aligned} I_{B2} &= I_{B1} - I_B \\ &= 0.1 - 0.01 = 0.09 \text{ mA} \end{aligned}$$

The required value of R_{B2} can now be found as

$$R_{B2} = \frac{V_B}{I_{B2}} = \frac{3.7}{0.09} = 41.1 \text{ k}\Omega$$

6.10

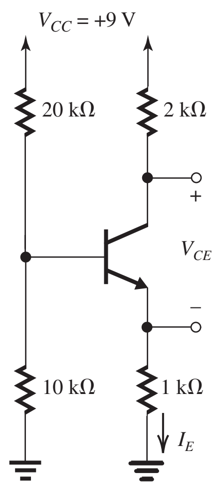

Figure 6.10.1

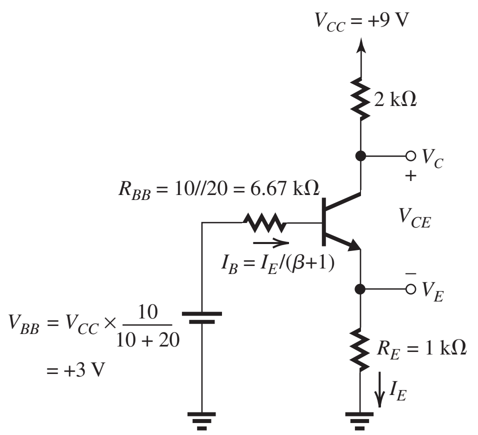

Figure 6.10.2

Using Thevenin's theorem, we can replace the voltage divider across V_{CC} with

$$V_{BB} = V_{CC} \times \frac{10}{10 + 20} = 9 \times \frac{10}{10 + 20} = +3 \text{ V}$$

and

$$R_{BB} = 10 \| 20 = 6.67 \text{ k}\Omega$$

The resulting circuit with this simplification is shown in Fig. 6.10.2. Noting that the current in the base is

$$I_B = \frac{I_E}{\beta + 1}$$

we can write a loop equation for the loop containing V_{BB}, R_{BB}, and the emitter circuit as

$$V_{BB} = \frac{I_E}{\beta + 1} R_{BB} + V_{BE} + I_E R_E$$

from which I_E can be found as

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_{BB}}{\beta + 1}}$$

Substituting $V_{BB} = 3$ V, $V_{BE} = 0.7$ V, and $R_E = 1$ kΩ, we obtain

$$I_E = \frac{3 - 0.7}{1 + \frac{6.67}{\beta + 1}} == \frac{2.3}{1 + \frac{6.67}{\beta + 1}} \tag{1}$$

The voltage V_E can then be found as

$$V_E = I_E R_E = I_E \times 1 = I_E \tag{2}$$

The collector current I_C is obtained as

$$I_C = \alpha I_E = \frac{\beta}{\beta + 1} I_E \tag{3}$$

and the collector voltage is found as

$$V_C = V_{CC} - I_C R_C = 9 - 2 I_C \tag{4}$$

Finally, V_{CE} can be calculated from

$$V_{CE} = V_C - V_E \tag{5}$$

Using Eqs. (1)–(5), we can obtain the following results for the three β values specified:

Case	β	I_E(mA)	V_E(V)	I_C(mA)	V_C(V)	V_{CE}(V)
a	∞	2.3	+2.3	2.3	+4.4	+2.1
b	100	2.16	+2.16	2.14	+4.72	+2.56
c	10	1.43	+1.43	1.30	+6.40	+4.97

Observe that in all cases V_{CE} is greater than 0.3 V, confirming that the transistor is operating in the active mode, as implicitly assumed.

6.11

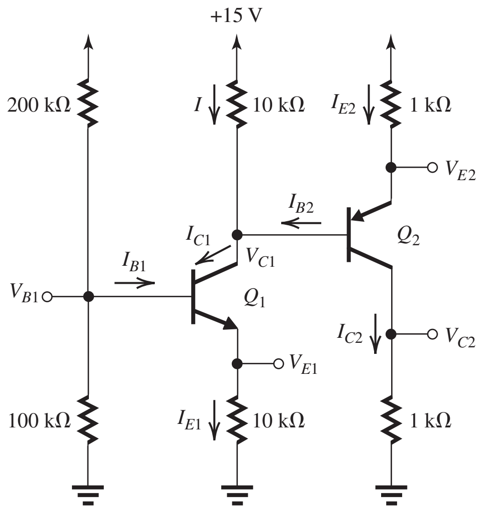

Figure 6.11.1

(a) $\beta = \infty$. Assume acive-mode operation for both transistors. The sequence of analysis steps is as follows:

(1) $I_{B1} = 0$
(2) $V_{B1} = 15 \times \frac{100}{100+200} = +5$ V.
(3) $V_{E1} = V_{B1} - V_{BE1} = 5 - 0.7 = +4.3$ V
(4) $I_{E1} = \frac{V_{E1}}{10\text{ k}\Omega} = \frac{4.3}{10} = 0.43$ mA
(5) $I_{C1} = \alpha_1 I_{E1} = 1 \times 0.43 = 0.43$ mA
(6) $I_{B2} = 0$
(7) $I = I_{C1} = 0.43$ mA
(8) $V_{C1} = 15 - 0.43 \times 10 = +10.7$ V
(9) $V_{E2} = V_{C1} + V_{EB2} = 10.7 + 0.7 = +11.4$ V
(10) $I_{E2} = \frac{15 - V_{E2}}{1\text{ k}\Omega} = \frac{15-11.4}{1} = 3.6$ mA
(11) $I_{C2} = \alpha_2 \times I_{E2} = 1 \times 3.6 = 3.6$ mA
(12) $V_{C2} = I_{C2} \times 1 = 3.6 \times 1 = +3.6$ V

Check: (1) $V_{C1} = +10.7$ V is higher than $V_{B1} = +5$ V, verifying that Q_1 is in the active mode. (2) $V_{C2} = +3.6$ V is lower than $V_{B2} = V_{C1} = +10.7$ V, verifying that Q_2 is in the active mode.

(b) Refer to Figure 6.11.2 below.

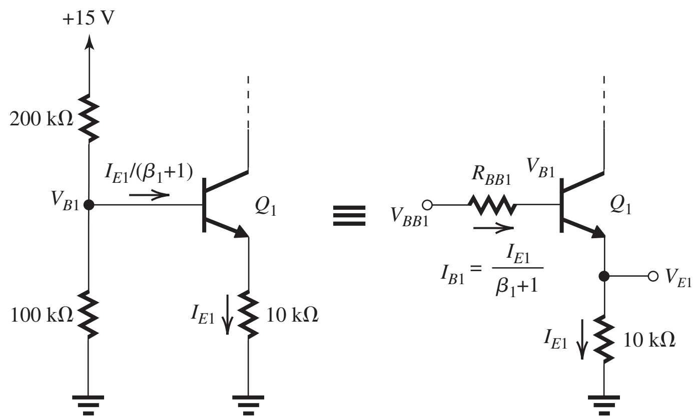

Figure 6.11.2

$\beta = 100$. Assume both transistors are operating in the active mode. To simplify the analysis, we replace the voltage divider that is connected to the base of Q_1 with its Thevenin equivalent, as shown in Fig. 6.11.2, where

$$V_{BB1} = 15 \times \frac{100}{100+200} = +5\ \text{V}$$

and

$$R_{BB1} = 100\ \text{k}\Omega \| 200\ \text{k}\Omega = 66.7\ \text{k}\Omega$$

Writing a loop equation for the base-emitter circuit of Q_1 enables us to find I_{E1} as

$$I_{E1} = \frac{V_{BB1} - V_{BE1}}{10 + \frac{R_{BB1}}{\beta_1+1}} = \frac{5 - 0.7}{10 + \frac{66.7}{100+1}} = 0.403\ \text{mA}$$

Continuing with the analysis:

$$I_{B1} = \frac{I_{E1}}{\beta_1+1} = \frac{0.403}{101} = 4\ \mu\text{A}$$

$$V_{E1} = I_{E1} \times 10 = +4.03\ \text{V}$$

$$I_{C1} = \alpha_1 I_{E1} = \frac{\beta_1}{\beta_1+1} I_{E1} = 0.99 \times 0.403$$
$$= 0.4\ \text{mA}$$

The remainder of the analysis can be simplified by replacing the circuit that is connected to the base of Q_2 by its Thevenin equivalent, as shown in Fig. 6.11.3 (refer Figure below).

Here, we have considered the collector of Q_1 as a constant-current source I_{C1}. Thus,

$$V_{BB2} = 15 - I_{C1} \times 10 = 15 - 0.4 \times 10 = 11\ \text{V}$$

$$R_{BB2} = 10\ \text{k}\Omega$$

Writing a loop equation for the base-emitter circuit of Q_2, we can obtain I_{E2} as

$$I_{E2} = \frac{15 - V_{EB2} - V_{BB2}}{1 + \frac{R_{BB2}}{\beta_2+1}}$$
$$= \frac{15 - 0.7 - 11}{1 + \frac{10}{101}} = 3\ \text{mA}$$

Continuing with the analysis:

$$V_{E2} = 15 - I_{E2} \times 1 = +12\ \text{V}$$

$$V_{C1} = V_{E2} - V_{EB2} = 12 - 0.7 = +11.3\ \text{V}$$

$$I_{B2} = \frac{I_{E2}}{\beta_2+1} = \frac{3}{101} = 0.03\ \text{mA}$$

$$I = I_{C1} - I_{B2} = 0.4 - 0.03 = 0.37\ \text{mA}$$

$$I_{C2} = \alpha_2 I_{E2} = 0.99 \times 3 = 2.97\ \text{mA}$$

$$V_{C2} = I_{C2} \times 1 = 2.97\ \text{V}$$

Check: $V_{C1}(+11.3\ \text{V}) > V_{B1}(+4.73\ \text{V})$, thus Q_1 is in active mode; and $V_{C2}(+2.97\ \text{V}) < V_{B2}(+11.3\ \text{V})$, thus Q_2 is in active mode.

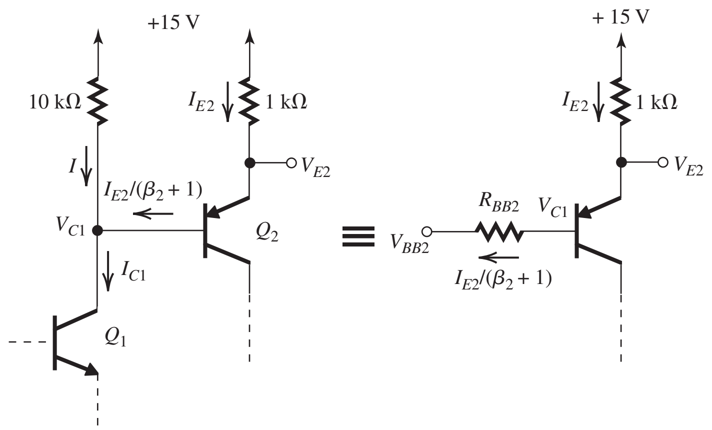

Figure 6.11.3

As a summary, Fig. 6.11.4 shows the results for the case $\beta = \infty$, and Fig. 6.11.5 shows the results for the case $\beta = 100$.

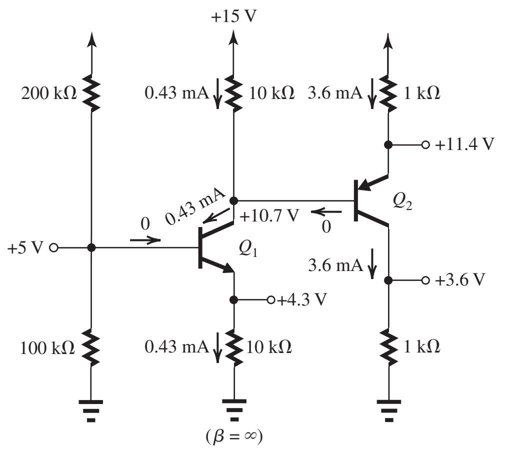

Figure 6.11.4

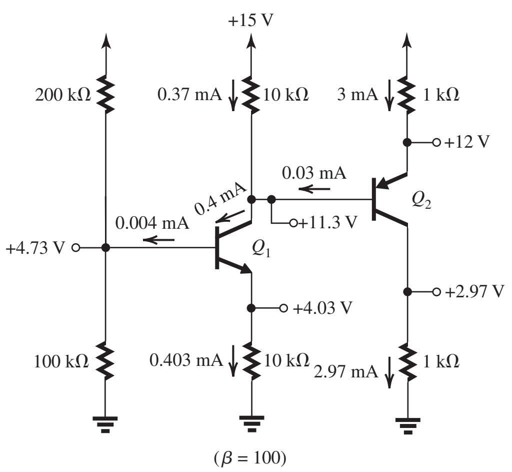

Figure 6.11.5

6.12

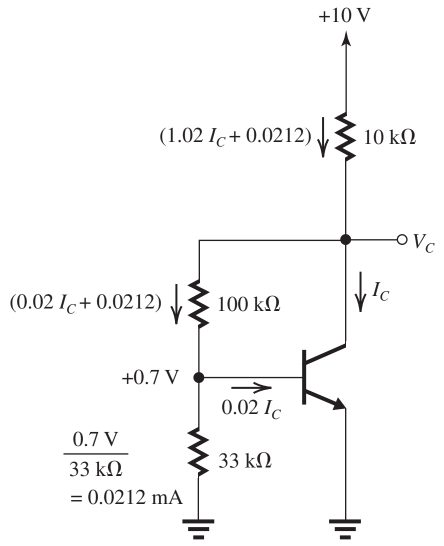

Figure 6.12.2

Figure 6.12.2 shows the circuit with most of the analysis. Here, since V_C is greater than V_B by the voltage drop across the 100-kΩ resistor, the transistor will be operating in the active mode. Our analysis assumed a collector current I_C and determined the base current as $I_C/\beta = I_C/50 = 0.02I_C$. The 33-kΩ resistor has a voltage across it equal to V_{BE}, that is, 0.7 V; thus, its current is $0.7/3.3 = 0.0212$ mA. A node equation at the base yields the current through the 100-kΩ resistor, and a node equation at the collector provides the current through the 10–kΩ resistor.

Now, writing an equation for the voltage between the supply (+10 V) to ground, we obtain

$$10 = 10(1.02I_C + 0.0212) + 100(0.02I_C + 0.0212) + 0.7$$

This equation can be solved to obtain

$$I_C = 0.571 \text{ mA}$$

Finally, the voltage V_C can be found from

$$V_C = 10 - (1.02I_C + 0.0212) \times 10$$
$$= 10 - 10.2 \times 0.571 - 0.212$$
$$V_C = 3.96 \text{ V}$$