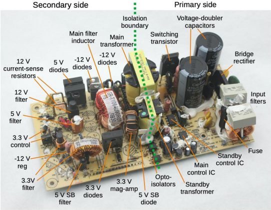


# Finite output resistance in saturation

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## Quick review

The equation used to define iD depends on relationship between $v_{DS}$ and $v_{OV}$:

- When $v_{DS} \ll v_{OV}$ (i.e., the small $v_{DS}$ model)

$$i_D = \left[ \left( \mu_n C_{ox} \left( \frac{W}{L} \right) v_{OV} \right) \right] v_{DS} \tag{1}$$

- When $v_{DS} < v_{OV}$ (i.e., the large $v_{DS}$ model)

$$\begin{aligned} I_D &= \mu_n C_{ox} \left( \frac{W}{L} \right) \left[ V_{OV} - \frac{1}{2} v_{DS} \right] v_{DS} \\ &= k_n' \left( \frac{W}{L} \right) \left[ V_{OV} - \frac{1}{2} v_{DS} \right] v_{DS} \end{aligned} \tag{2}$$

- When $v_{DS} \geq v_{OV}$ (channel pinch-off and current saturation)

$$i_D = \frac{1}{2} k_n' \left( \frac{W}{L} \right) \tag{3}$$

- But what would happen when $v_{DS} \gg v_{OV}$?



## Finite output resistance in saturation

- In previous lectures, we assume (in saturation) iD is independent of $v_{DS}$.
- Therefore, a change in $v_{DS}$ has no effect on $i_D$.
  - This implies that the incremental resistance $R_S$ is infinite
  - It is based on the idealization that, once the n-channel is pinched off, changes in $v_{DS}$ will have no effect on $i_D$.
  - The problem is that, in practice, this is not completely true.
- In reality, the drift current increases, and $i_D$ increases with increasing $v_{DS}$



## Finite Output Resistance in Saturation

What effect does increasing $v_{DS}$ has on the n-channel once pinch-off has occurred?

- It will cause the pinch-off point to move slightly away from the drain and create new depletion region.
- Voltage across the (now shorter) channel will remain at $v_{OV}$.
- However, the additional voltage applied at $v_{DS}$ will be seen across the "new" depletion region.

# Finite Output Resistance in Saturation

What effect will increased $v_{DS}$ has on n-channel once pinch-off has occurred?

- This voltage accelerates electrons as they reach the drain end, and sweep them across the "new" depletion region.
- However, at the same time, the length of the n-channel will decrease. This is known as channel length modulation.



# Finite output resistance in saturation

- When $v_{DS} > V_{OV}^2$, the depletion region around the drain region grows in size.
- With depletion-layer widening, the channel length is in effect reduced, from $L$ to $L - \Delta L$ , a phenomenon known as channel-length modulation.

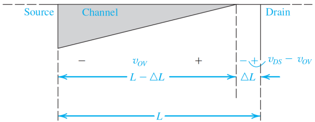


**FIG 1. Early Effect—Finite Output Resistance**
increasing $v_{DS}$ beyond $v_{DSsat}$ causes the channel pinch-off point to move



# Finite output resistance in saturation

- As the channel length becomes shorter, the electric field, which is proportional to $vDS/L$, becomes larger.
- Since $i_D$ is inversely proportional to the channel length, $i_D$ increases with $v_{DS}$.

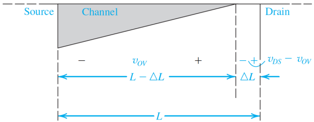


**FIG 2. Early Effect—Finite Output Resistance**
increasing $v_{DS}$ beyond $v_{DSsat}$ causes the channel pinch-off point to move slightly away from the drain; thus, reduces the effective channel lengthy by $\Delta L$



# Finite output resistance in saturation

- In reality, the drift current increases, and $i_D$ increases with increasing $v_{DS}$

$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2(1 + \lambda v \quad (4)$$

- $\lambda$ is a device parameter with the units of $V^{-1}$, the value of which depends on manufacturer's design and manufacturing process. $\lambda$ is much larger for newer tech's
- The value of $\lambda$ depends both on

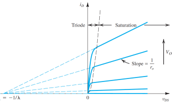


**FIG 3. Early Effect—Finite Output Resistance**
Effect of $v_{DS}$ on $i_D$ in the saturation region. The MOSFET parameter VA depends on the process technology and, for a given process, is proportional to the channel length L.

# Finite output resistance in saturation

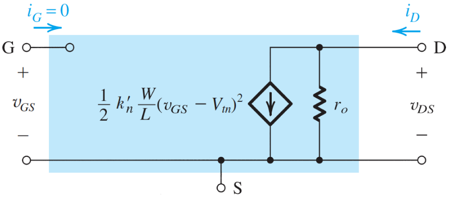


**FIG 4.** Large-Signal Equivalent Model of the n-channel MOSFET in saturation, incorporating the output resistance $r_o$. The output resistance models the linear dependence of $i_D$ on $v_{DS}$ and is given by Equation (4). Please note the addition of finite output resistance $r_o$.



# Defining the output resistance

Note that $r_o$ is the 1/slope of $i_D$ vs $v_{DS}$ curve

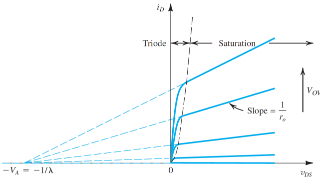


**FIG 5. Early Effect—Finite Output Resistance**
Effect of $v_{DS}$ on $i_D$ in the saturation region. The MOSFET parameter VA depends on the process technology and, for a given process, is proportional to the channel length L.



# Defining the output resistance

- Note that $r_o$ is the 1/slope of $i_D$ vs $v_{DS}$ curve

$$r_o \equiv \left[\frac{\partial i_D}{\partial v_{DS}}\right]^{-1} \tag{5}$$

- Combining Equation (4) and Equation (5), we have

$$\begin{aligned}\frac{\partial i_D}{\partial v_{DS}} &= \frac{\partial}{\partial v_{DS}}\frac{1}{2}k'_n\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2(1 + \lambda v_{DS}) \\ &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}v_{ov}^2\lambda\end{aligned} \tag{6}$$

- Thus, the output resistor is defined as shown in Equation (7)

$$\begin{aligned}r_o &= \left[\lambda\frac{k'_n}{2}\frac{W}{L}(V_{GS} - V_{tn})^2\right]^{-1} \\ &= \frac{1}{\lambda i_D} \\ &= \frac{V_A}{i_D}\end{aligned} \tag{7}$$



# The end